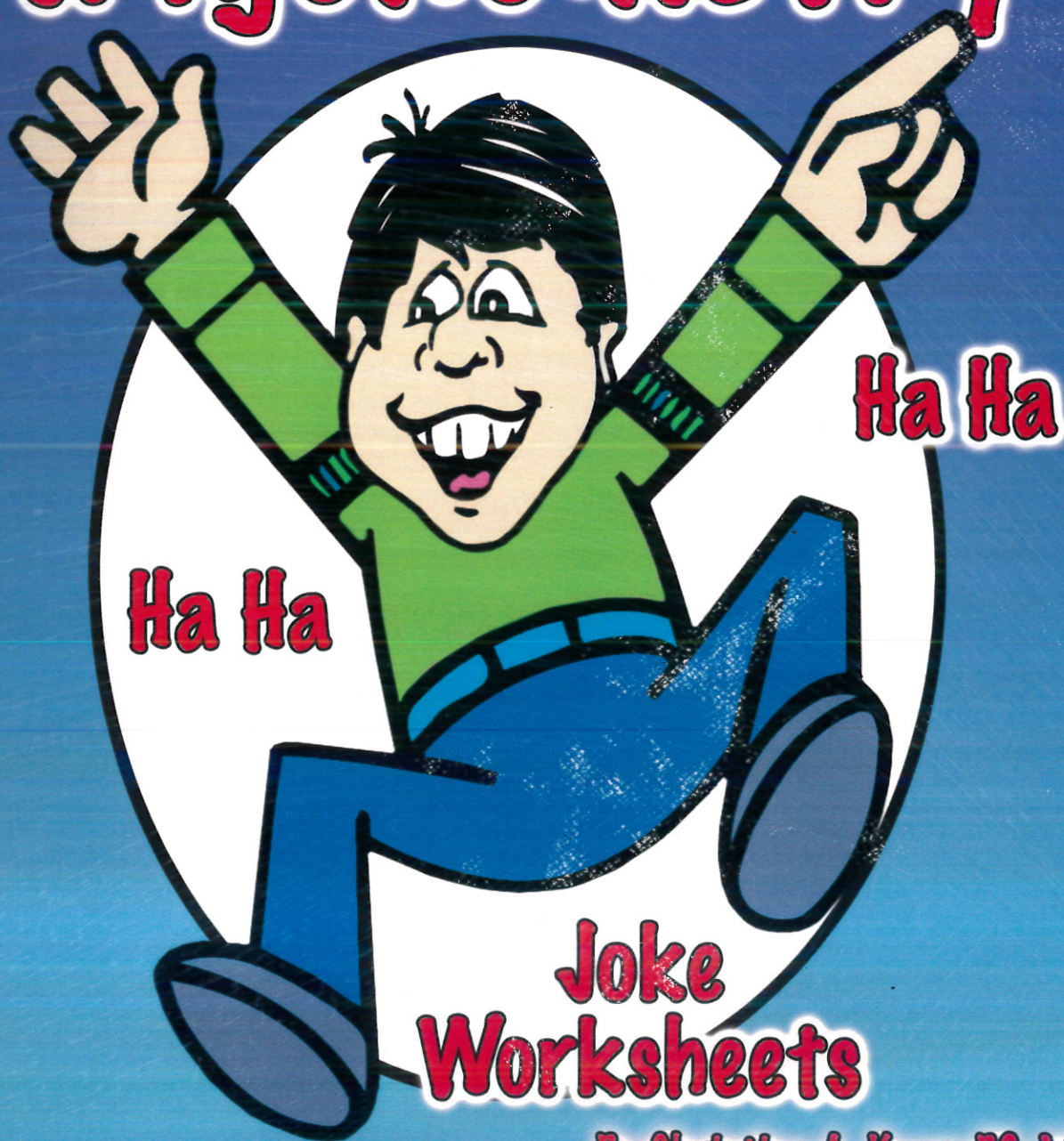


Nasco's Dedicated to Delivery

Trigonometry



By Christine A. Koers BS, MS

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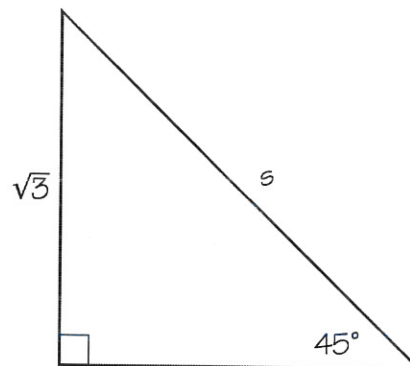
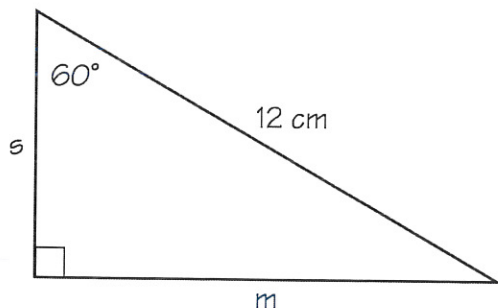
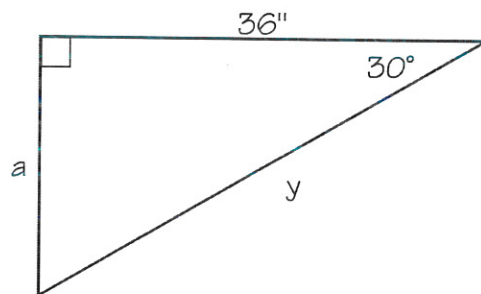
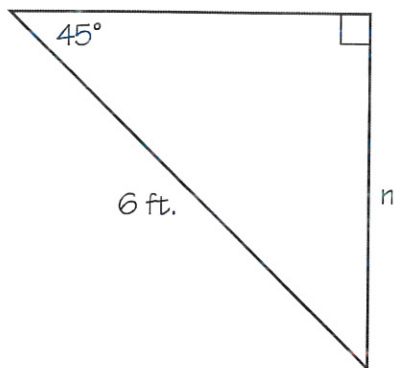
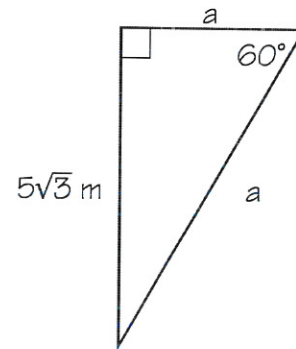
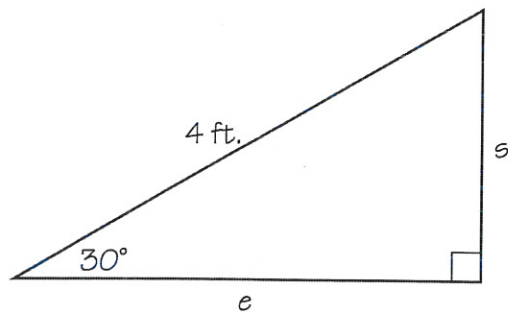
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"What is the favorite game on Navy ships?"

Solve the following missing lengths. Put the letter of the missing length above the answer line below. When all the letters are filled in, the joke will be solved.



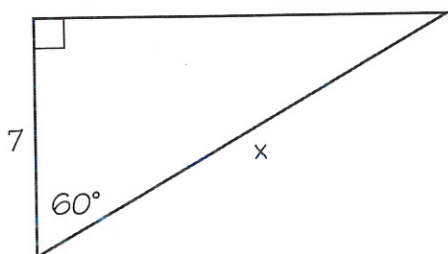
$\sqrt{6}$ $2\sqrt{3}$ $12\sqrt{3}$ $6\sqrt{3}$ 10 $3\sqrt{2}$

6 5 $24\sqrt{3}$ 2

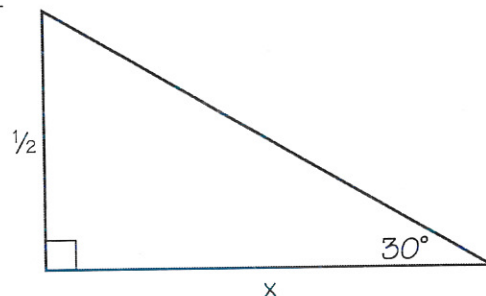
"What do you have if you have had the same mustard before?"

Find the missing lengths. The answer to each problem will match a letter that will allow you to figure out the joke.

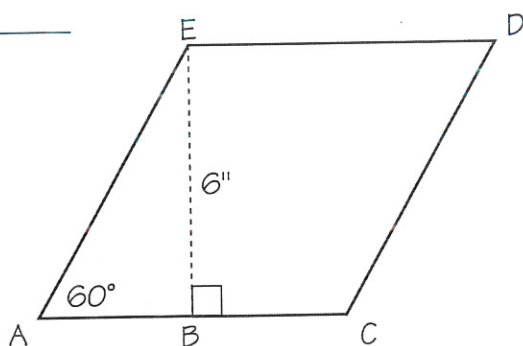
1. $x =$ _____



2. $x =$ _____

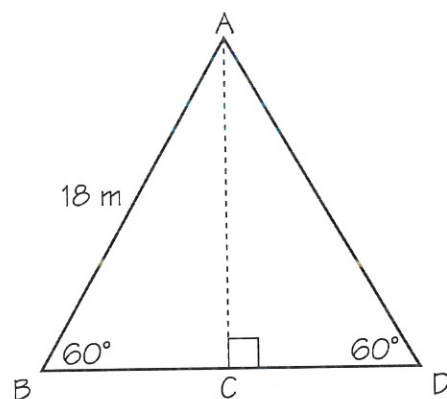


3. $AE =$ _____

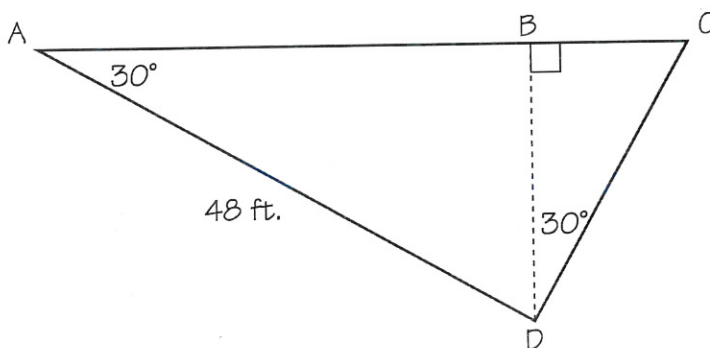


4. $AC =$ _____

5. $CD =$ _____



6. $BC =$ _____



7. $CD =$ _____

A: 8

N: $9\sqrt{3}$

J: 9

D: $16\sqrt{3}$

O: 14

E: $\sqrt{3}$

U: $4\sqrt{3}$

V: $8\sqrt{3}$

C: $\frac{1}{2}$

B: $5\sqrt{3}$

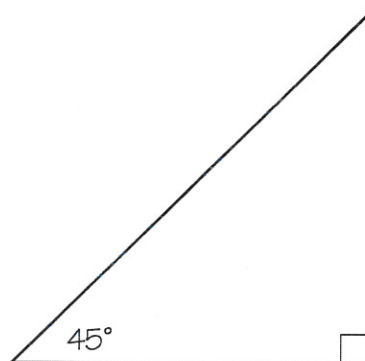
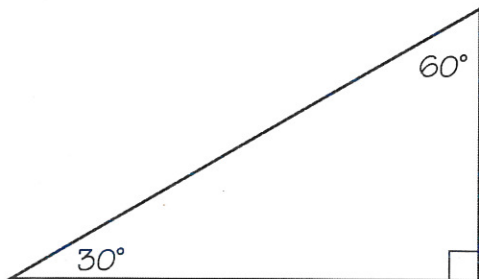
I: $\frac{\sqrt{3}}{2}$

W: 3

7 2 5 1 4

6 3

Special Triangles - Study Sheet



	0°	30°	45°	60°	90°
sin					
cos					

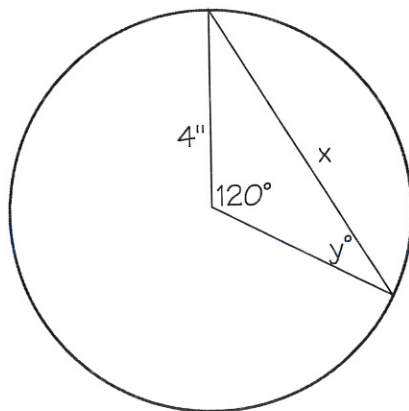
	0°	30°	45°	60°	90°
tan					
cot					

	0°	30°	45°	60°	90°
sec					
csc					

"What happens when the smog lifts in Los Angeles?"

Solve the following word problems. Cross out the letter that matches your answer. The remaining letters will allow you to figure out the joke.

- A guy wire is attached to a pole and makes an angle of 60° with the ground. The telephone pole is 33 ft. tall.
 - Draw a picture of this situation.
 - What is the length of the wire?
 - Find the distance on the ground between the pole and the guy wire.
- A 20-ft. ladder is placed against a house so that its base is 10 ft. from the house.
 - Draw a picture of this situation.
 - Find the angle the ladder makes with the ground.
 - Find the exact height where the ladder meets the building.
- Use the drawing below to find the following. (Hint: use the altitude of an isosceles triangle to help.)
 - Find x .
 - Find angle Y .



F	S	U	A	C	D	O	L	E	A
$22\sqrt{3}$	30	$\sqrt{3}$	$4\sqrt{3}$	$2\sqrt{3}$	$11\sqrt{3}$	$10\sqrt{3}$	$12\sqrt{3}$	60	11

Answer: _____

"If you have seen one shopping center, then you have..."

Solve the following word problems. The answer to each problem will match a letter that will allow you to figure out the joke.

1. An equilateral triangle has a height of 15".

I: $\sqrt{3}$

a. Draw a picture of this situation.

T: 12

b. What is the length of each side?

A: $21\sqrt{3}$

c. What is the area of the triangle?

M: 6

2. A person 63" tall casts a shadow with a 60° angle of elevation towards the sun.

N: $10\sqrt{3}$

a. Draw a picture of this situation.

D: 8

b. What is the length of the shadow?

S: 90

L: $75\sqrt{3}$

3. Use the drawing to find the following. ACDE is a square. EC is 12" long.

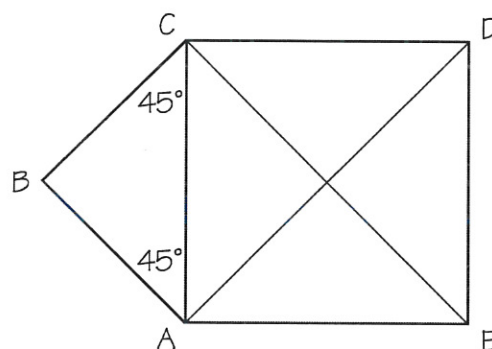
O: $\sqrt{2}$

a. AE = _____

E: $6\sqrt{2}$

b. AB = _____

c. Area of ABCDE = _____



3c

3a

3a

1b

2b

3b

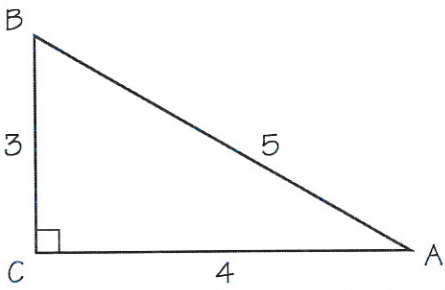
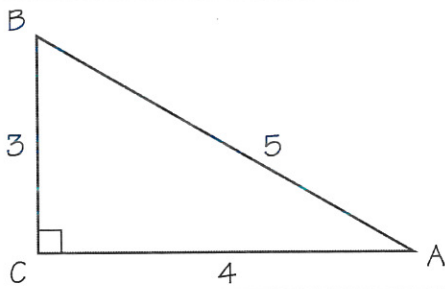
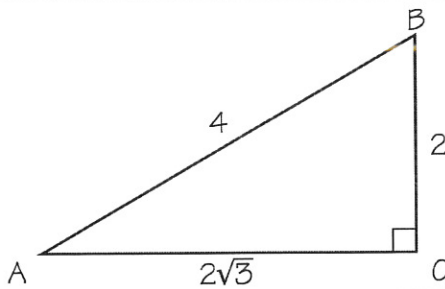
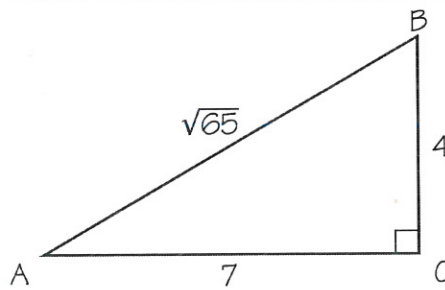
2b

1c

1c

"Did you hear about the man who lost his left side?"

Select the correct ratio from the four choices given. Write the letter of the correct choice on the line below to figure out the joke.

1. $\sin A$ 2. $\cos A$ 3. $\tan A$		C: $\frac{4}{3}$ W: $\frac{4}{5}$	O: $\frac{3}{4}$ S: $\frac{3}{5}$
4. $\sin B$ 5. $\cos B$ 6. $\tan B$		E: $\frac{3}{5}$ N: $\frac{4}{5}$	O: $\frac{5}{3}$ A: $\frac{4}{3}$
7. $\sin A$ 8. $\cos A$ 9. $\tan A$		H: $\frac{1}{2}$ B: $\sqrt{3}$	T: $\frac{\sqrt{3}}{2}$ I: $\frac{\sqrt{3}}{3}$
10. $\sin B$ 11. $\cos B$ 12. $\tan B$		U: $\frac{4}{7}$ L: $\frac{7\sqrt{65}}{65}$	R: $\frac{7}{4}$ G: $\frac{4\sqrt{65}}{65}$

$\frac{\quad}{7}$ $\frac{\quad}{5}$ $\frac{\quad}{1}$ $\frac{\quad}{6}$ $\frac{\quad}{10}$ $\frac{\quad}{10}$
 $\frac{\quad}{12}$ $\frac{\quad}{9}$ $\frac{\quad}{11}$ $\frac{\quad}{7}$ $\frac{\quad}{8}$ $\frac{\quad}{4}$ $\frac{\quad}{3}$ $\frac{\quad}{2}$

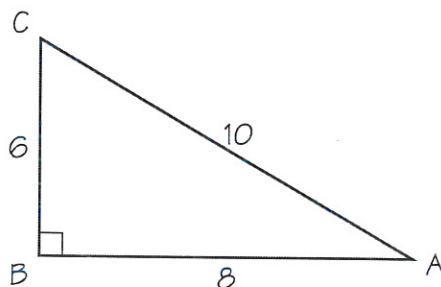
"What did Bob say to the acupuncturist?"

Select the correct ratio from the four choices given. Write the letter of the correct choice on the line below to figure out the joke.

1. $\tan C$

2. $\sin A$

3. $\cos A$



O: $\frac{3}{5}$

N: $\frac{4}{5}$

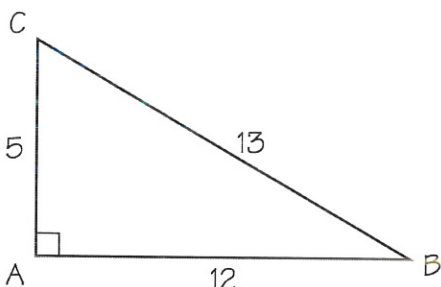
Y: $\frac{5}{4}$

A: $\frac{4}{3}$

4. $\cos B$

5. $\sin B$

6. $\tan C$



L: $\frac{5}{13}$

E: $\frac{12}{5}$

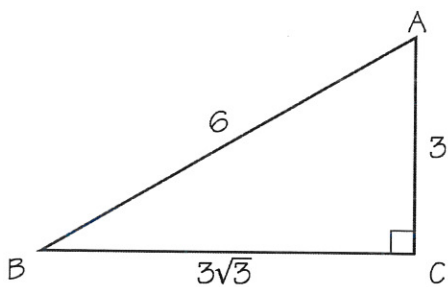
B: $\frac{12}{13}$

U: $\frac{5}{12}$

7. $\sin A$

8. $\cos A$

9. $\tan A$



W: $\frac{1}{2}$

V: 2

J: $\sqrt{3}$

D: $\frac{\sqrt{3}}{2}$

9

1

4

8

6

5

5

7

2

3

6

"To have a successful diet, you must have..."

Select the correct ratio. The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\sin A$

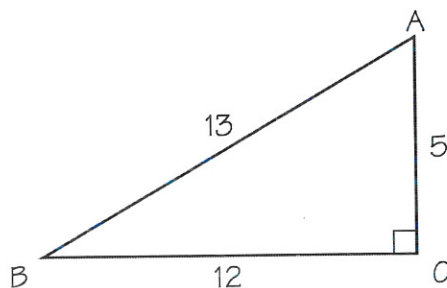
2. $\cos A$

3. $\tan A$

4. $\csc A$

5. $\sec A$

6. $\cot A$



O: $\frac{12}{5}$

A: $\frac{5}{13}$

U: 1

E: $\frac{13}{5}$

B: $\frac{2}{5}$

I: $\frac{13}{12}$

D: $\frac{12}{13}$

N: $\frac{5}{12}$

7. $\sec B$

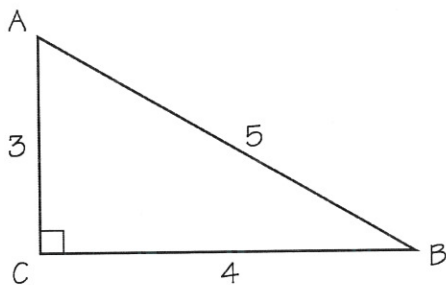
8. $\tan A$

9. $\cos A$

10. $\cot A$

11. $\sin A$

12. $\csc B$



P: $\frac{5}{3}$

V: $\frac{3}{5}$

M: $\frac{4}{5}$

S: $\frac{1}{4}$

T: $\frac{5}{4}$

R: $\frac{4}{3}$

W: 0

L: $\frac{3}{4}$

11 4 6 1

3 9 5 8

12 10 2 7 7 5 8

"What do you call two people in an ambulance?"

Select the correct ratio. The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\sin B$

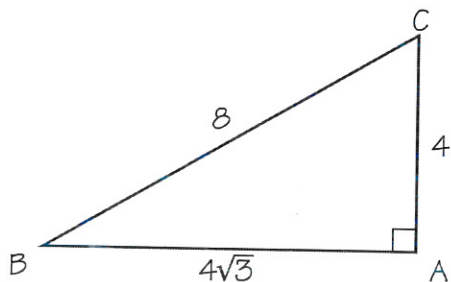
2. $\cos B$

3. $\tan C$

4. $\csc B$

5. $\sec B$

6. $\cot C$



O: $\frac{\sqrt{3}}{2}$

A: $\frac{1}{2}$

R: $\sqrt{3}$

E: $\frac{2\sqrt{3}}{3}$

S: 2

T: $\sqrt{6}$

U: $\frac{1}{3}$

A: $\frac{\sqrt{3}}{3}$

7. $\sec A$

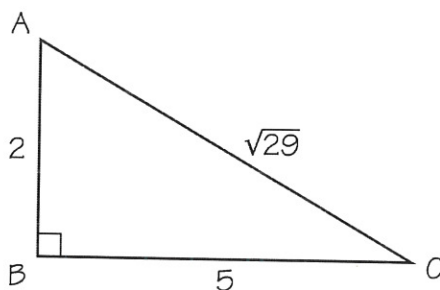
8. $\cot C$

9. $\sin A$

10. $\csc A$

11. $\tan C$

12. $\cos A$



C: $\frac{5\sqrt{29}}{29}$

D: $\frac{\sqrt{29}}{5}$

W: $\sqrt{29}$

M: $\frac{2\sqrt{29}}{29}$

I: $\frac{5}{2}$

P: $\frac{2}{5}$

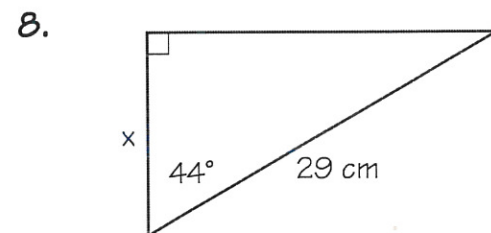
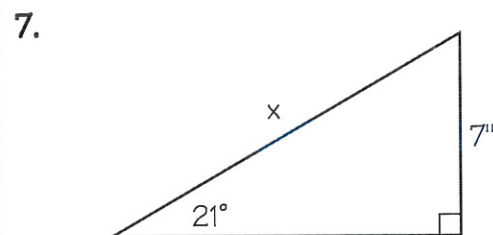
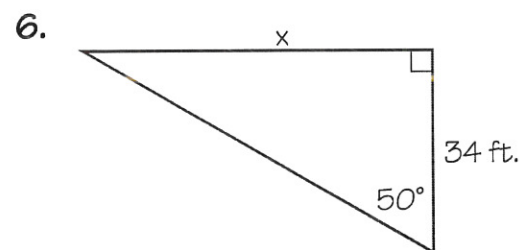
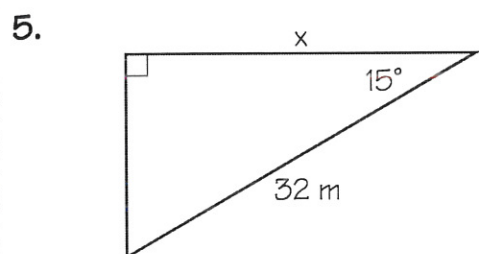
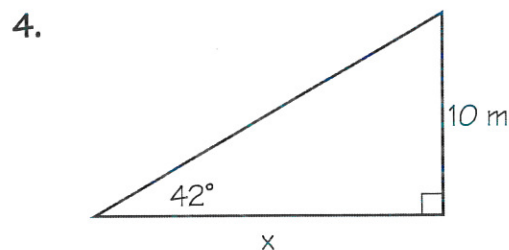
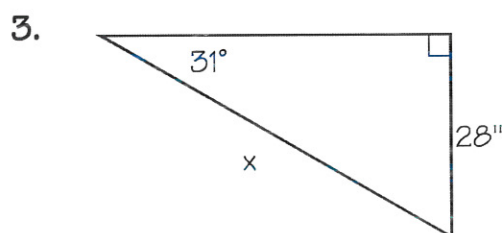
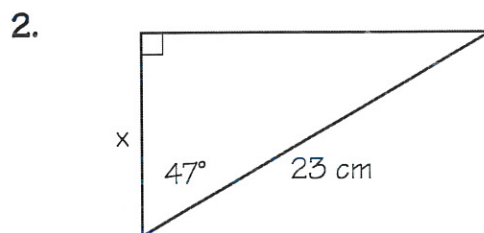
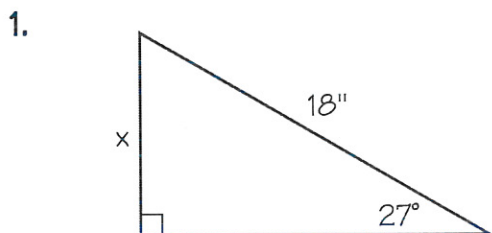
F: $\frac{\sqrt{29}}{2}$

Y: $\frac{1}{5}$

1 11 6 8 3 2 7 12 5 10 8 9 4

"What is a pessimist's blood type?"

Solve the following missing lengths. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.



S: 12.3

B: 20.9

V: 30.9

G: 11.1

U: 50.6

A: 8.2

T: 15.7

O: 6.5

N: 19.5

R: 10.7

I: 54.4

W: 0.3

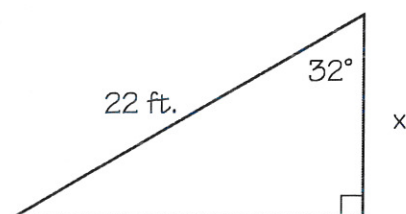
E: 40.5

8 7 6 4 2 1 3 5 6

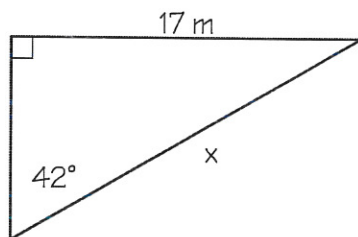
"Where do erasers go for vacation?"

Solve the following missing lengths. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

1.



2.



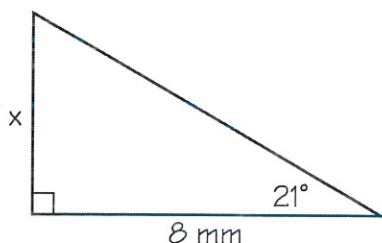
Z: 17.4

T: 3.1

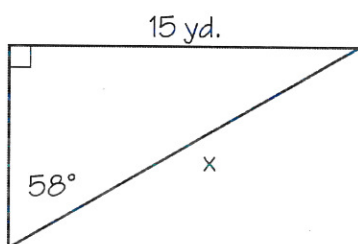
U: 4.8

S: 22.6

3.



4.



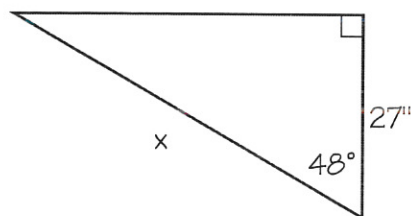
N: 17.7

R: 25.4

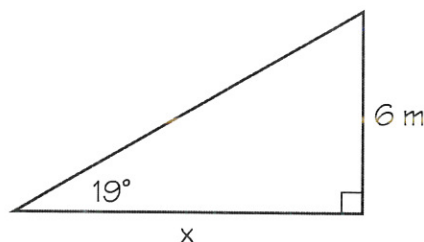
I: 12.2

A: 40.4

5.



6.



E: 35.5

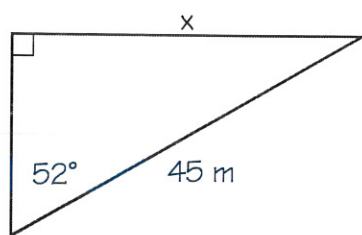
B: 10.8

D: 37.4

J: 37.9

O: 18.7

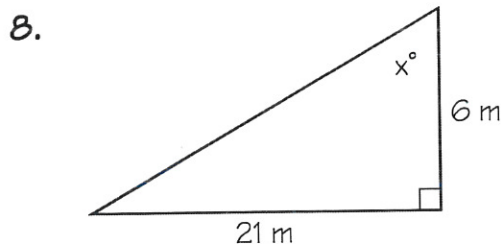
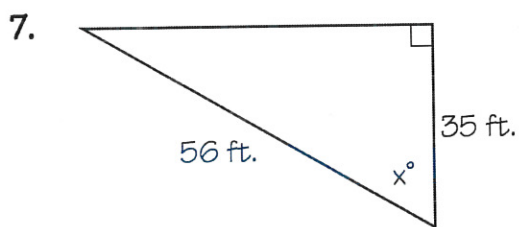
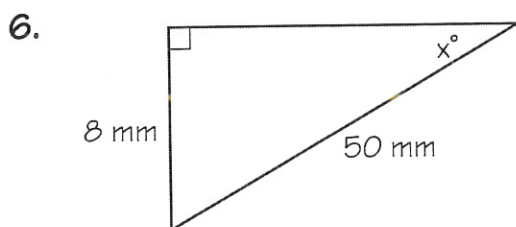
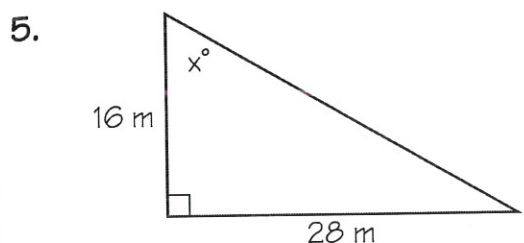
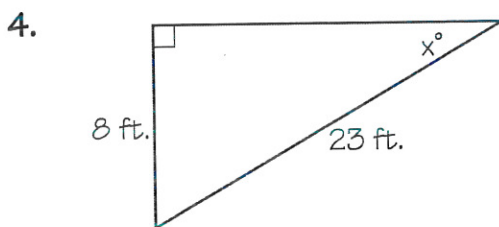
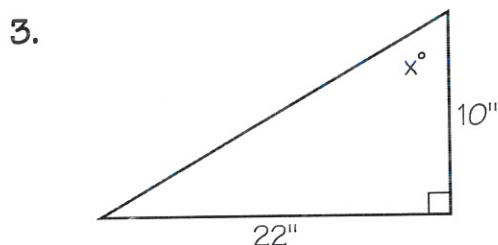
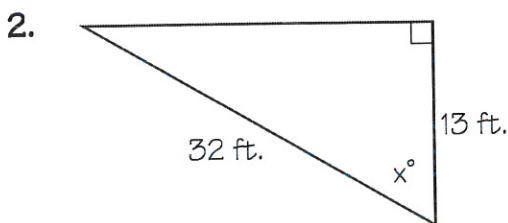
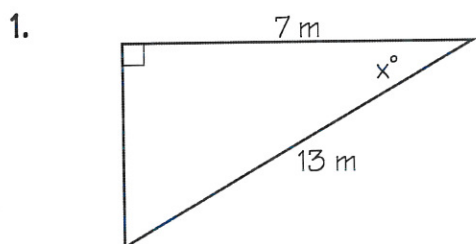
7.



3 1 7 2 5 6 1 4 5

"What type of hair-do do sea captains hate?"

Solve the following missing angles. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.



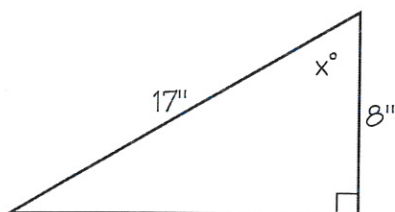
- W: 51.3°
- A: 17.5°
- S: 57.4°
- C: 74.1°
- D: 35.1°
- T: 60.3°
- R: 20.4°
- V: 47.2°
- U: 65.6°
- B: 11.1°
- C: 9.2°
- E: 66.0°
- K: 0.9°

8 4 2 7 6 3 5 1

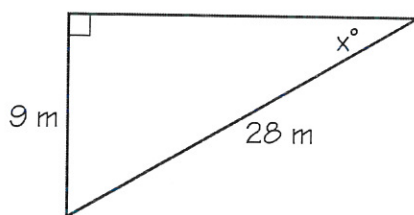
"What happens when a ghost gets lost in a fog?"

Solve the following missing angles. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.

1.



2.



M: 30.9°

R: 27.6°

S: 61.9°

E: 65.2°

D: 40.7°

I: 18.7°

T: 73.5°

A: 19.3°

S: 24.9°

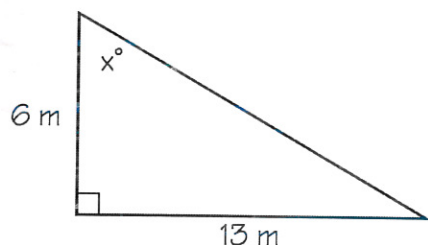
H: 48.2°

O: 67.3°

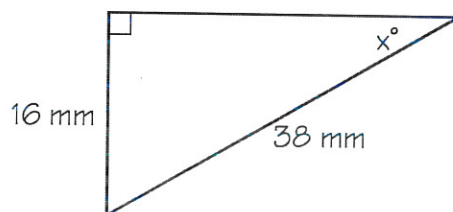
C: 12.3°

W: 44.5°

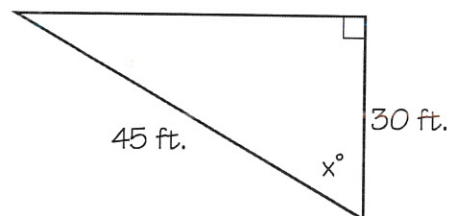
3.



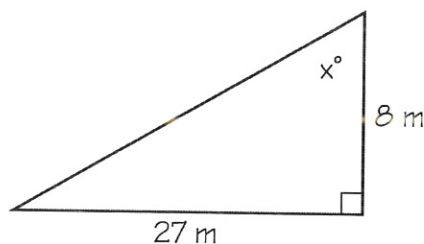
4.



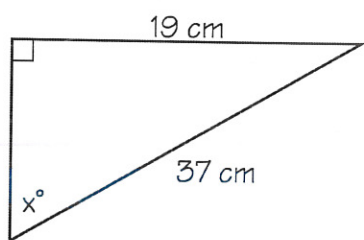
5.



6.



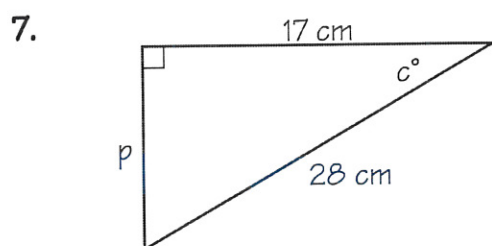
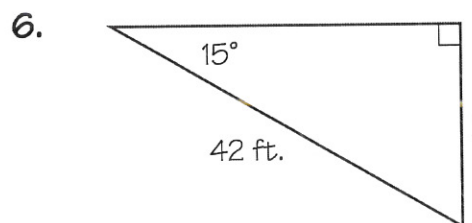
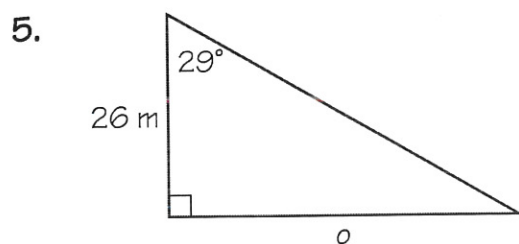
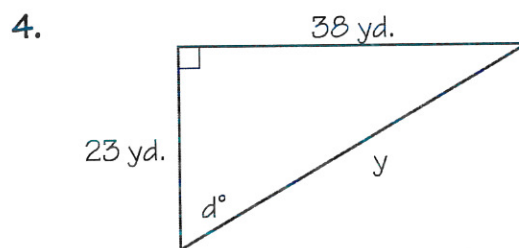
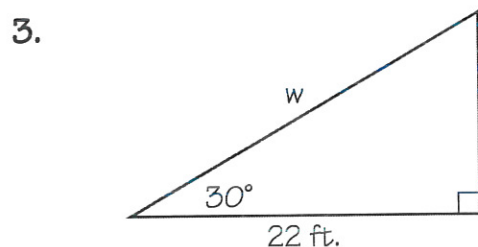
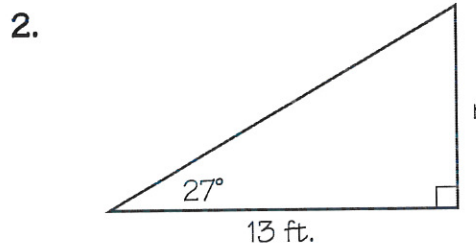
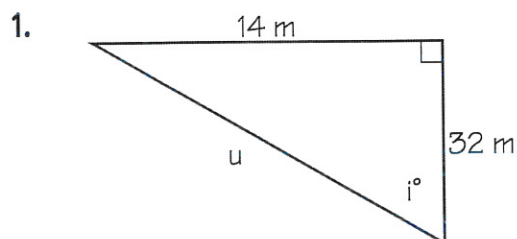
7.



5 3 1 7 2 4 6

"What makes headlines?"

Solve the following missing lengths and angles. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.

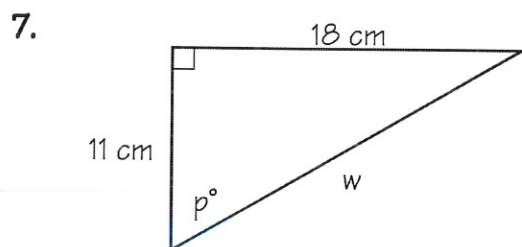
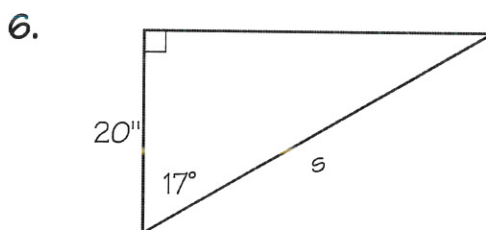
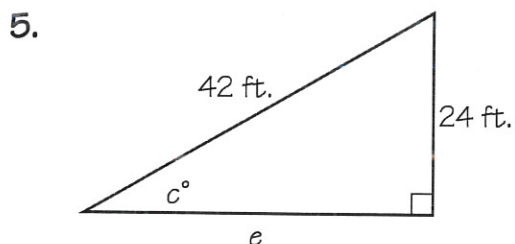
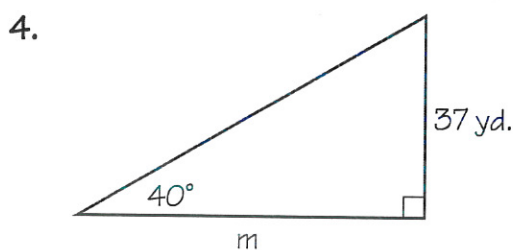
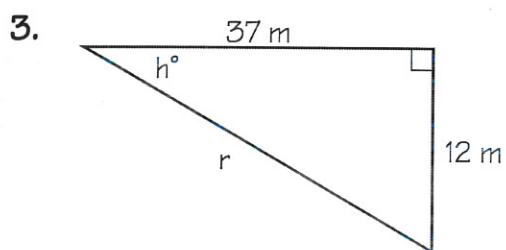
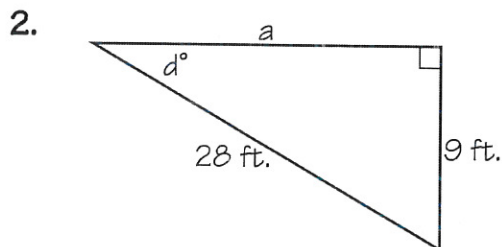
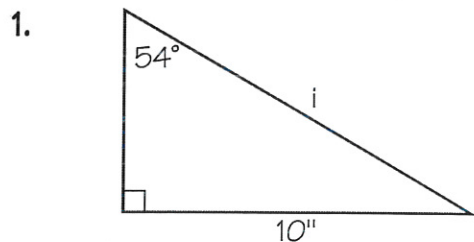


28.2 52.6 14.4 6.6 58.8 34.9 6.6 14.4 44.4 13.8

22.2 23.6 10.9 10.9 14.4 25.4 37.6 18.3

"What tops off a ghost's sundae?"

Solve the following missing lengths and angles. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.



21.1 18.0 12.4 58.6 58.6 34.5 18.7

20.9 34.8 38.9 34.5 26.5 44.1

"How does a backward poet write?"

Solve the following word problems. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.

1. A building's angle of elevation from a point on the ground 60 ft. from its base is 32° . What is the height of the building?

T: 66.3

E: 202.3

A: 101.6

2. A 16-ft. ladder is leaning against a house. The ladder's angle of elevation is 47° . How far above the ground is the top of the ladder?

I: 16.9

S: 37.5

W: 153.2

3. What is the angle of elevation of the sun when a 57" person casts a 36" shadow?

N: 57.7

U: 73.9

4. A balloon has an angle of elevation of 54° from a point on the ground. From the point on the ground to the balloon, the distance is 250 ft. How far up in the air is the balloon?

R: 11.7

P: 45.5

V: 14.5

D: 10.1

5. A kite is 50 m high when 200 m of string is out. What angle does the kite string make with the ground?

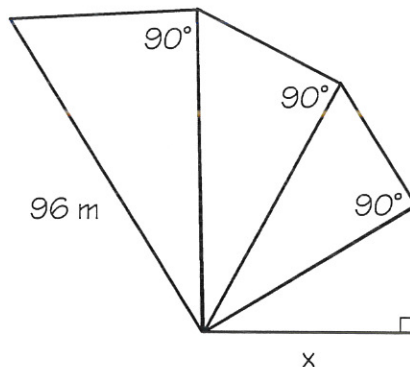
6. A table shaped like a parallelogram has sides of 3 ft. and 6 ft. One angle on the table is 70° . Find the area of the table.

6 3 5 4 2 1 4

"Who is always putting everyone down?"

Solve the following word problems. Round your answers to the nearest tenth. Cross out the letter that matches your answer. The remaining letters will allow you to figure out the joke.

1. How far from the base of a house is the bottom of a 25-ft. ladder that makes an angle of 65° with the ground?
2. If a person 6 ft. tall has a shadow of 54", then what is the angle of elevation of the sun?
3. The following triangles are all $30^\circ - 60^\circ - 90^\circ$ triangles. Find the length of x .



4. From a point 200 yds. from the base of a cliff, the angles of elevation to the top and bottom of a lighthouse on top of the cliff are 60° and 58° . How tall is the lighthouse?

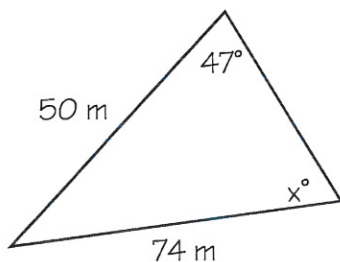
B	G	E	R	A	V	N	I	T	S	Y
26.3	18.4	54	63.2	10	8.4	10.6	9.5	67.3	53.1	7.4

Answer: _____

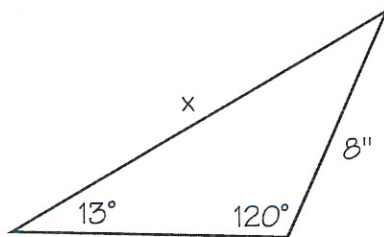
"Why do ghosts go to baseball games?"

Solve for the missing lengths and angles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

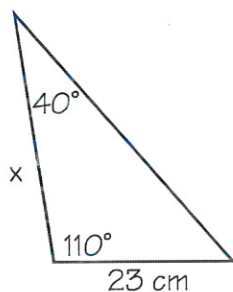
1.



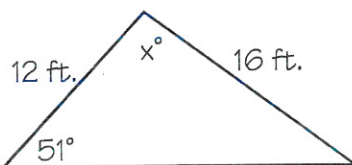
2.



3.



4.



5. $a = 18$, $c = 13$, $\angle A = 32^\circ$, $\angle C =$ _____

6. $c = 22$, $\angle B = 48^\circ$, $\angle C = 62^\circ$, $b =$ _____

7. $a = 12$, $b = 10$, $\angle A = 43^\circ$, $\angle B =$ _____

8. $a = 31$, $b = 38$, $\angle A = 30^\circ$, $\angle B =$ _____

9. $b = 27$, $\angle A = 50^\circ$, $\angle B = 75^\circ$, $c =$ _____

10. $a = 5$, $b = 8$, $\angle B = 38^\circ$, $\angle C =$ _____

O: 119.4

A: 71.3

H: 30.8

B: 34.6

S: 11.5

M: 29.6

N: 22.3/157.7

I: 17.9

T: 22.9

R: 93.3

E: 37.8/142.2

W: 65.5

P: 22.5

G: 15.4/164.6

U: 18.5

Y: 73.8

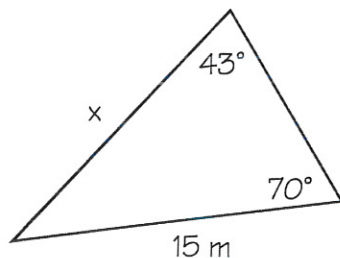
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 6 1 5 3 4 8

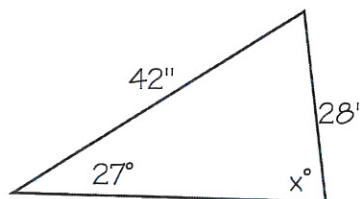
"What kinds of tests do they give witches?"

Solve for the missing lengths and angles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

1.



2.



E: 42.8

R: 12.9

T: 23.5

N: 19.8

Y: 54.8

M: 20.7

H: 22.7

I: 35.4

W: 46

S: 42.9

A: 119.3

U: 84.2

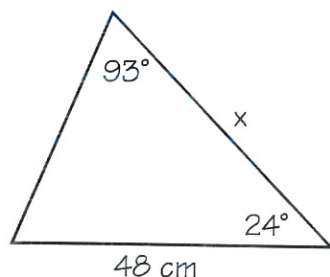
X: 36.8

P: 90.3

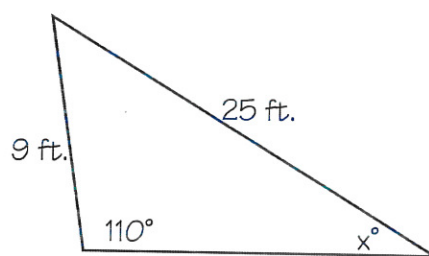
O: 38.5

L: 17.4

3.



4.



5. $c = 46$, $\angle B = 47^\circ$, $\angle C = 61^\circ$, $b =$ _____

6. $a = 15$, $c = 11$, $\angle A = 33^\circ$, $\angle C =$ _____

7. $a = 34$, $b = 42$, $\angle A = 29^\circ$, $\angle B =$ _____

8. $a = 15$, $b = 13$, $\angle A = 42^\circ$, $\angle B =$ _____

9. $a = 6$, $b = 9$, $\angle B = 37^\circ$, $\angle C =$ _____

10. $b = 26$, $\angle A = 49^\circ$, $\angle B = 74^\circ$, $c =$ _____

10 3 7 9 1 8 4 9 6 8 5 4 2

“What do you call a person who jumps off a Paris bridge?”

Solve the following word problems. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

1. A baseball pennant is in the shape of an isosceles triangle. The base is 18" long. The sides meet at an angle of 32° . How long are the sides?

K: 340.2

E: 34.7

N: 32.7

L: 2,460.6

S: 8.8

O: 43.9

P: 199.2

E: 21.6

Y: 75.6

I: 2,731.7

T: 4.9

W: 30.5

2. A guy wire attached to a telephone pole makes a 67° angle with the ground. At a point 20 ft. farther out from the guy wire, the angle of elevation of the top of the pole is 35° . How long is the guy wire?

3. A fire is sighted from two ranger stations that are 4,000 m apart. The angles of observation to the fire measure 50° from one station and 43° from the other station. Find the distance along the line of sight to the fire from the closer of the two stations.

4. A loading ramp 6 m long makes a 24° angle with the level ground beneath it. The ramp is replaced by another ramp 16 m long. Find the angle that the new ramp makes with the ground.

4 3 2 1 2

"What type of crime is it when you steal someone's coffee?"

Solve the following word problems. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.

1. Two angles of a triangle measure 31° and 54° . The longest side is 48 cm. Find the length of the shortest side.

W: 75.9

A: 143.6

N: 15,605.1

R: 3.5

U: 5.2

S: 53.7

G: 125.9

A: 10,340.3

I: 24.8

F: 63.7

M: 29.1

E: 83.4

P: 22.9

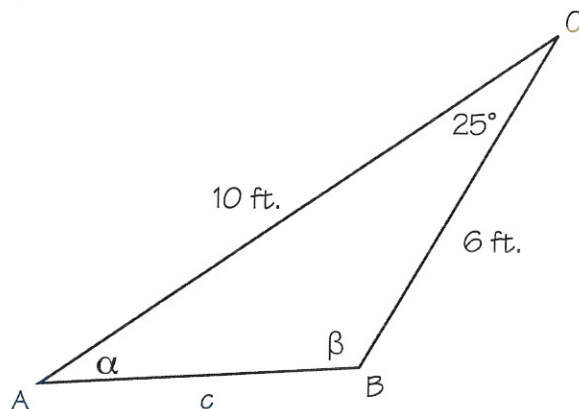
2. A hang glider will land on a 4,500-ft. clearing in a field. From the hang glider, the angle of depression of the opposite ends of the field measure 25° and 32° . How far is the hang glider from the nearest end of the field?

Solve triangle ABC shown below. (Hint: Draw a perpendicular line from A to the line through $\overline{BC}$.)

3. $c =$ _____

4. $\alpha =$ _____

5. $\beta =$ _____



4

3

5

5

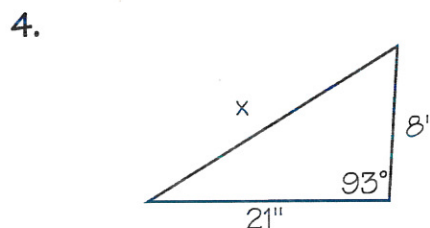
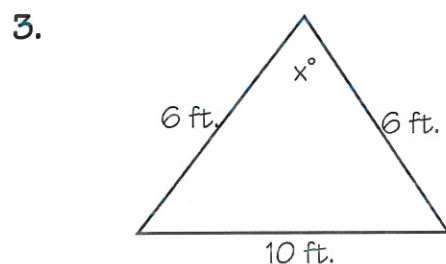
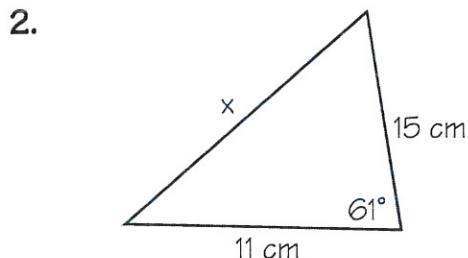
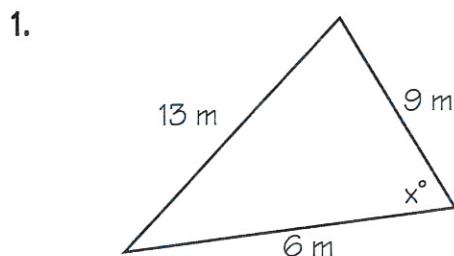
1

2

5

"What do prisoners use to call each other?"

Solve for the missing lengths and angles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.



T: 24.7

P: 16.5

A: 18.3

N: 13.6

K: 110.5

H: 7.8

B: 44.7

C: 33.1

S: 22.9

U: 123.1

L: 43.9

E: 112.9

D: 64.5

O: 118.8

W: 42.6

5. $a = 10, b = 12, c = 17, \angle B =$ _____

6. $a = 5, c = 9, \angle B = 60^\circ, b =$ _____

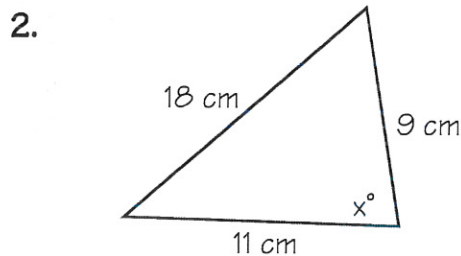
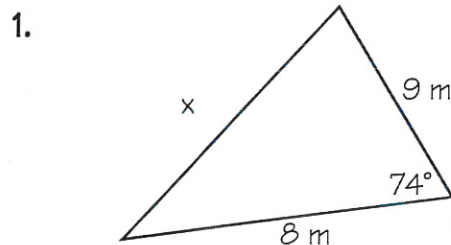
7. $a = 20, b = 8, c = 14, \angle C =$ _____

8. $b = 17, c = 6, \angle A = 75^\circ, a =$ _____

7 3 5 5 8 6 1 2 3 4

"What did the nuclear physicist have for lunch?"

Solve for the missing lengths and angles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.



F: 19.1

A: 43.2

E: 23.8

S: 34.2

H: 8.5

D: 14.4

O: 10.3

G: 85.5

P: 128.0

K: 101.4

C: 29.3

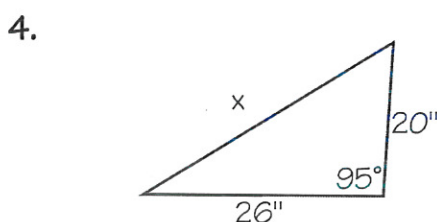
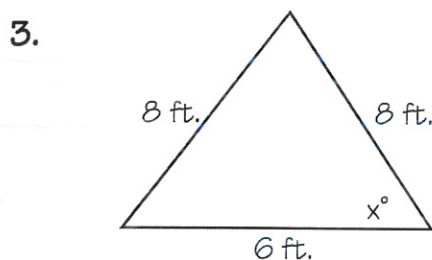
N: 68.0

I: 36.9

M: 75.1

S: 15.2

L: 54.9

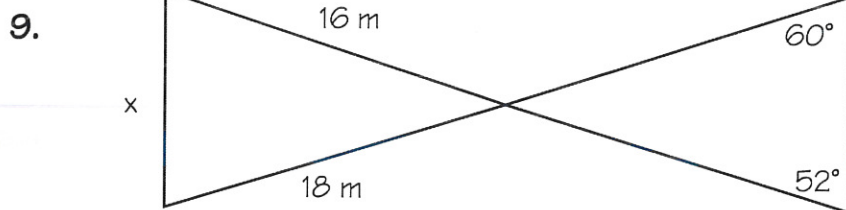


5. $a = 6, c = 10, \angle B = 58^\circ, b = \underline{\hspace{2cm}}$

6. $a = 12, b = 16, c = 20, \angle A = \underline{\hspace{2cm}}$

7. $a = 22, b = 11, c = 17, \angle B = \underline{\hspace{2cm}}$

8. $b = 15, c = 6, \angle A = 80^\circ, a = \underline{\hspace{2cm}}$



9

6

8

8

6

1

3

7

5

6

2

4

"What kind of street does a ghost like best?"

Solve the following word problems. Round your answers to the nearest tenth.
The answer to each problem will match a letter that will allow you to figure out the joke.

- Two airplanes leave the same airport at the same time. The first plane flies 170 km/h in a direction of 300° . The second plane flies 210 km/h in a direction of 220° . After two hours, how far apart are the planes?
I: 435.6
M: 45.7
O: 34.1
D: 492.3
- A piece of wire 6.2 ft. long is bent into a triangular shape. One side is 1.7 ft. long and another is 2.4 ft. long. Find the angles of the triangle.
J: 305.4
E: 31.2
N: 42.5
- A pee wee baseball diamond is a square, each side being 60 ft. long. The pitcher's mound is 40 ft. from home plate. How far is the pitcher's mound to first base?
S: 73.5
42.6
63.9
D: 296.4
Y: 56.8
- What is the perimeter of an equilateral triangle inscribed in a circle with a radius of 6"?
A: 77.5
43.8
58.7
W: 291.1
- Plane A has been flying for an hour and a half at 400 mph heading 130° from the Indianapolis International Airport. Plane B has been flying for one hour at 350 mph heading 150° from the same airport. How far apart are the two planes?

2 5 4 2 1 4 3 5

"What do you call a mathematics teacher's retirement?"

Solve the following word problems. Round your answers to the nearest tenth. Cross out the letter that matches your answer. The remaining letters will allow you to figure out the joke.

1. Two planes leave an airport at the same time. One plane is flying 250 mph, and the other plane is flying 325 mph. The angle between their flight paths is 65° . After two hours, how far apart are they?
2. An isosceles triangle has a vertex angle of 36° . If the two congruent sides are 22" long, what is the area of the triangle?
3. The distance from home plate to dead center is 300 ft. If the distance between the bases is 60 ft., how far is it from dead center to third base?
4. The sides of a parallelogram are 15 cm and 23 cm, and one angle is 42° . Find the length of the longer diagonal.

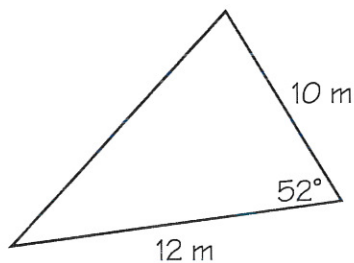
M	A	F	B	T	E	R	W	M	Y	A	T	H
35.6	10.5	36.1	261	100.3	0.5	53.4	142.1	7.1	630.7	8.3	123.6	5.6

Answer: _____

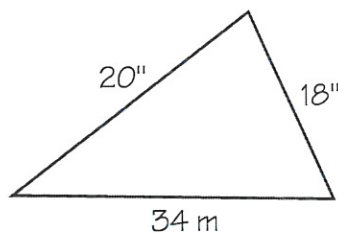
"What lies at the bottom of the ocean and twitches?"

Find the area of the triangles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

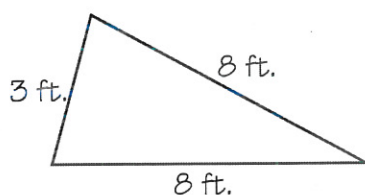
1.



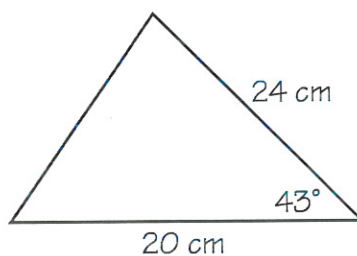
2.



3.



4.



U: 82.7

I: 74.1

V: 47.3

T: 154.0

W: 81.3

A: 102.4

D: 24.9

S: 144.0

K: 86.2

Y: 19.5

N: 103.6

O: 11.8

J: 105.8

R: 163.7

E: 22.4

C: 12.2

5. $a = 14, b = 12, \angle C = 100^\circ$

6. $a = 13, b = 24, c = 15$

7. $a = 5, b = 9, c = 10$

8. $b = 13, c = 20, \angle A = 52^\circ$

9. $a = 23, c = 17, \angle B = 32^\circ$

10. $a = 34, b = 8, c = 28$

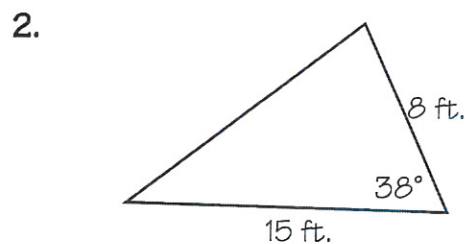
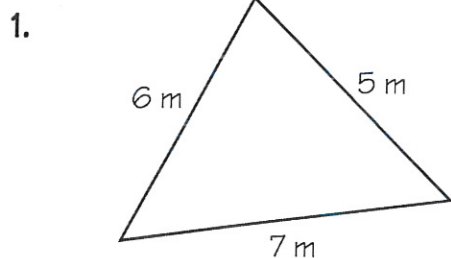
11. $a = 3, b = 12, c = 10$

8 9 7 4 1 3 5 2

10 4 7 11 6

"Why did the baby ghost go to the doctor before Halloween?"

Find the area of the triangles. Round your answers to the nearest tenth. The answer to each problem will match a letter that will allow you to figure out the joke.

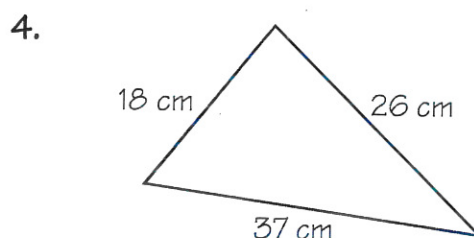
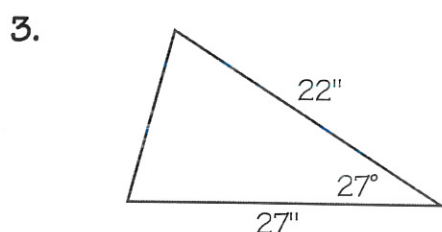


O: 134.8

T: 15.2

W: 33.4

J: 19.3



A: 36.9

B: 226.9

C: 132.6

R: 14.7

5. $a = 23, b = 14, c = 12$

G: 69.5

6. $a = 8, b = 5, \angle C = 92^\circ$

E: 215.0

7. $b = 17, c = 23, \angle A = 57^\circ$

V: 165.4

8. $a = 4, b = 10, c = 8$

H: 20.0

9. $a = 25, c = 20, \angle B = 35^\circ$

K: 31.2

10. $a = 40, b = 24, c = 22$

O: 143.4

S: 164.0

8 3 5 4 8 2

10 9 9 7 8 4 1 7 6 9 8

"What do you call Peruvian Rorschach tests?"

Find the exact value for the following. Rationalize any radical denominator.
The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\sin 60^\circ$

B: $-\frac{\sqrt{2}}{2}$

2. $\cos 120^\circ$

N: $-\frac{1}{2}$

3. $\sin 150^\circ$

R: undefined

4. $\sin 180^\circ$

A: 1

I: -1

5. $\cos 360^\circ$

E: 2

S: $\frac{\sqrt{3}}{2}$

6. $\cos 315^\circ$

L: 0

7. $\sin 240^\circ$

D: $\sqrt{3}$

O: $\frac{1}{2}$

8. $\sin 585^\circ$

T: $\frac{\sqrt{2}}{2}$

9. $\cos 540^\circ$

C: $-\frac{\sqrt{3}}{2}$

9 2 7 5 8 4 3 6 1

"Why did Alex get fired from the tailor shop?"

Find the exact value for the following. Rationalize any radical denominator.
The answer to each problem will match a letter that will allow you to figure out the joke.

- | | |
|----------------------|--------------------------|
| 1. $\cos 60^\circ$ | D: 1 |
| 2. $\sin 120^\circ$ | O: 0 |
| 3. $\cos 240^\circ$ | E: $-\frac{\sqrt{2}}{2}$ |
| 4. $\cos 180^\circ$ | T: undefined |
| 5. $\sin 225^\circ$ | H: $-\frac{1}{2}$ |
| 6. $\cos -270^\circ$ | B: -1 |
| 7. $\sin 450^\circ$ | A: $\frac{\sqrt{3}}{2}$ |
| 8. $\cos 315^\circ$ | Y: $\sqrt{3}$ |
| 9. $\sin 600^\circ$ | I: $\frac{1}{2}$ |
| | S: $\frac{\sqrt{2}}{2}$ |
| | J: $-\frac{\sqrt{3}}{2}$ |
| | U: -2 |

<u>3</u>	<u>5</u>	<u>7</u>	<u>1</u>	<u>7</u>	<u>2</u>
<u>8</u>	<u>6</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>6</u>
					<u>4</u>

"What type of vacation do nuclear physicists go on?"

Find the exact value for the following. Rationalize any radical denominator.
The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\tan 45^\circ$

S: 0

2. $\cot 60^\circ$

T: $-\frac{\sqrt{3}}{3}$

3. $\cot 135^\circ$

O: 1

I: undefined

4. $\tan 330^\circ$

H: 3

5. $\tan 240^\circ$

N: -1

R: $\frac{\sqrt{3}}{3}$

6. $\cot 360^\circ$

P: $\sqrt{3}$

7. $\tan 540^\circ$

A: $\frac{1}{3}$

Y: -3

8. $\cot 150^\circ$

F: $-\sqrt{3}$

D: $-\sqrt{2}$

8 6 7 7 6 1 3 4 2 6 5

"What does an Olympic fencer do at noon each day?"

Find the exact value for the following. Rationalize any radical denominator. The answer to each problem will match a letter that will allow you to figure out the joke.

- | | |
|----------------------|--------------------------|
| 1. $\cot 30^\circ$ | T: undefined |
| 2. $\tan 135^\circ$ | E: -1 |
| 3. $\tan 180^\circ$ | C: 3 |
| 4. $\cot 240^\circ$ | O: $-\frac{\sqrt{3}}{3}$ |
| 5. $\tan 300^\circ$ | L: 1 |
| 6. $\cot 540^\circ$ | Y: 2 |
| 7. $\cot 480^\circ$ | N: $\frac{\sqrt{3}}{3}$ |
| 8. $\tan -135^\circ$ | U: $\sqrt{3}$ |
| | A: $-\frac{1}{3}$ |
| | O: 0 |
| | G: $-\sqrt{3}$ |
| | B: $-\sqrt{2}$ |

$\frac{5}{6}$ $\frac{3}{7}$ $\frac{7}{8}$ $\frac{1}{1}$ $\frac{6}{4}$ $\frac{5}{5}$ $\frac{3}{2}$

"What do Eskimos get from sitting on the ice too long?"

Find the exact value of μ between 0° and 180° for which each is true. The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\sin \mu = \frac{\sqrt{3}}{2}$

T: 40°

2. $\cos \mu = -\frac{1}{2}$

I: 180°

3. $\cos \mu = -1$

O: 150°

S: 60°

4. $\tan \mu = 1$

E: 110°

D: 135°

5. $\cos \mu = -\frac{\sqrt{2}}{2}$

P: 90°

A: 120°

6. $\sin \mu = \frac{1}{2}$

B: 70°

L: 45°

7. $\cos \mu = -\frac{\sqrt{3}}{2}$

R: 30°

U: 155°

8. $\tan \mu = \text{undefined}$

8 7 4 2 6 7 3 5 1

"What trail do crazy people take through the forest?"

Find the exact value of μ between 0° and 180° for which each is true. The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\cos \mu = \frac{1}{2}$

E: 130°

2. $\sin \mu = \frac{\sqrt{2}}{2}$

C: 45°

3. $\tan \mu = -1$

Y: 90°

D: 165°

4. $\cos \mu = -\frac{\sqrt{3}}{2}$

P: 120°

A: 60°

5. $\sin \mu = \frac{1}{2}$

R: 20°

T: 150°

6. $\cos \mu = -1$

H: 180°

B: 50°

7. $\tan \mu = \text{undefined}$

O: 135°

8. $\cos \mu = -\frac{1}{2}$

S: 30°

8

5

7

2

6

3

8

1

4

6

"What do you call it when you say one thing but mean your mother?"

Make the conversion from degrees to radians and vice versa. The answer to each problem will match a letter that will allow you to figure out the joke.

Convert from degrees to radians:

1. 40°

2. 100°

A: $-\frac{7\pi}{4}$

I: $\frac{5\pi}{9}$

3. -225°

4. 350°

E: $\frac{8\pi}{3}$

Y: $-\frac{\pi}{3}$

5. -315°

6. 480°

U: $\frac{2\pi}{9}$

N: $-\frac{5\pi}{4}$

I: $\frac{35\pi}{18}$

M: $\frac{2\pi}{3}$

Convert from radians to degrees:

7. $\frac{\pi}{3}$

8. $\frac{3\pi}{5}$

W: 110°

D: 108°

9. $-\frac{3\pi}{4}$

10. $\frac{11\pi}{6}$

R: -135°

P: 330°

11. $-\frac{7\pi}{3}$

12. $\frac{9\pi}{4}$

L: 60°

F: 405°

S: -420°

T: 90°

12

9

6

1

8

2

5

3

11

7

4

10

"Why did the golfer bring two pairs of pants to the game?"

Make the conversion from degrees to radians and vice versa. The answer to each problem will match a letter that will allow you to figure out the joke.

Convert from degrees to radians:

1. 80°

2. 110°

I: $-\frac{20\pi}{9}$

S: $\frac{4\pi}{9}$

3. -135°

4. 25°

E: $\frac{7\pi}{3}$

D: $\frac{\pi}{3}$

5. -400°

6. 420°

T: $\frac{5\pi}{36}$

A: $\frac{11\pi}{18}$

B: $\frac{15\pi}{8}$

L: $-\frac{3\pi}{4}$

Convert from radians to degrees:

7. $\frac{\pi}{4}$

8. $\frac{2\pi}{3}$

G: 120°

N: 210°

9. $-\frac{3\pi}{5}$

10. $\frac{11\pi}{3}$

H: -108°

O: 660°

11. $\frac{7\pi}{6}$

12. $-\frac{8\pi}{5}$

U: 130°

C: 45°

A: -288°

K: 70°

$\frac{5}{11}$

$\frac{7}{2} \frac{1}{6}$

$\frac{9}{6}$

$\frac{8}{10} \frac{4}{4}$

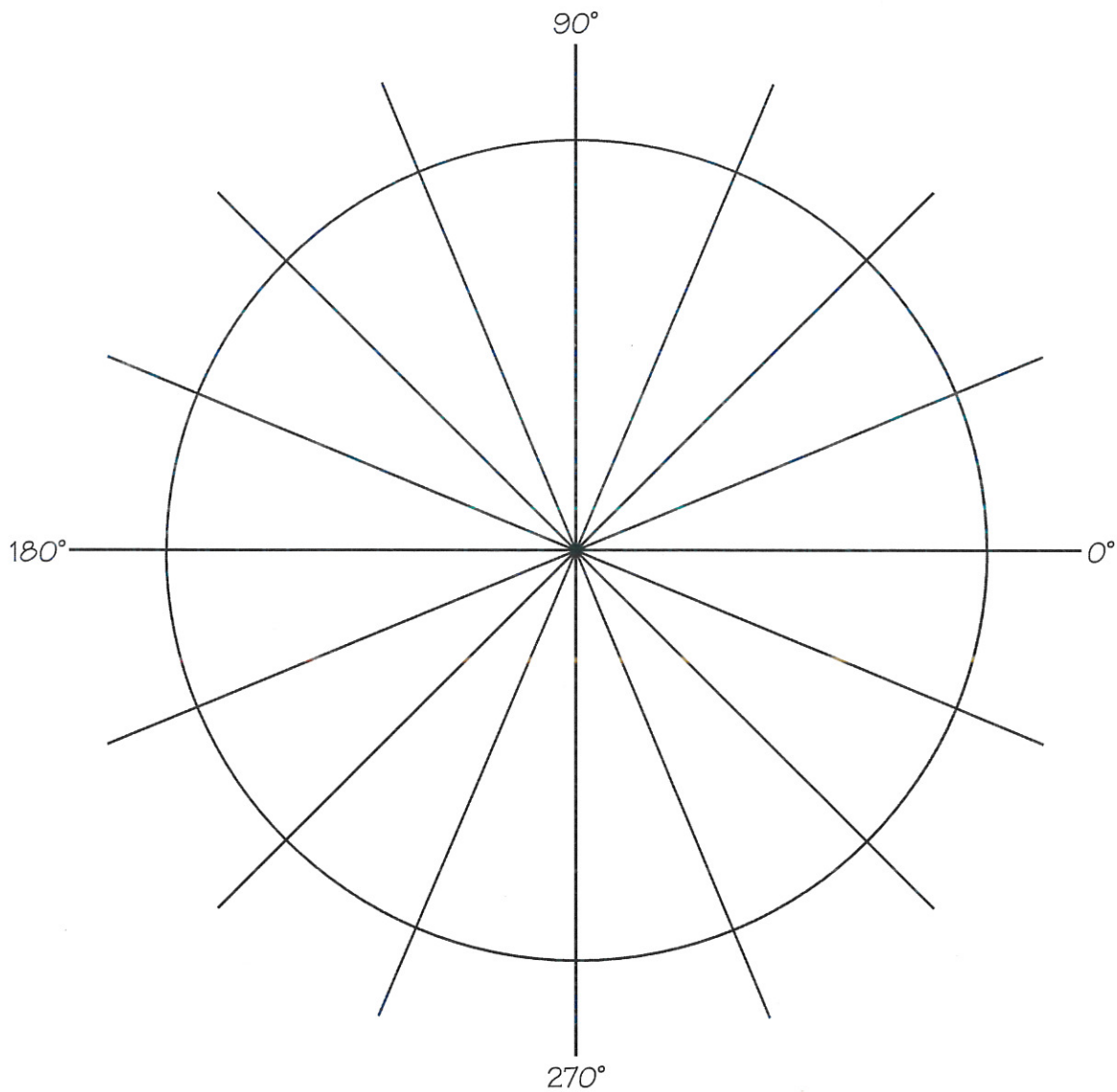
$\frac{12}{12}$

$\frac{9}{10} \frac{3}{6}$

$\frac{5}{11}$

$\frac{10}{11} \frac{6}{6}$

Unit Circle - Study Sheet



"What do you get from a pampered cow?"

Find the exact value for the following. Rationalize any radical denominator.
The answer to each problem will match a letter that will allow you to figure out the joke.

- | | |
|--|------------------------------|
| 1. $\sin 30^\circ + \cos 30^\circ$ | O: 1 |
| 2. $\sin 45^\circ - \cos 60^\circ$ | L: $\frac{1 + \sqrt{3}}{2}$ |
| 3. $\tan 30^\circ - \cos 180^\circ$ | D: 4 |
| 4. $2 \cos 120^\circ + 3 \sin (-30^\circ)$ | I: $-\frac{5}{2}$ |
| 5. $(\tan 45^\circ)(\cos 30^\circ)$ | S: $-\frac{\sqrt{3}}{6}$ |
| 6. $(\sec 60^\circ)(\csc 30^\circ)$ | M: 0 |
| 7. $\sin \frac{\pi}{6} + \cos \frac{\pi}{3}$ | P: $2\sqrt{2}$ |
| 8. $\csc \frac{\pi}{4} + \sec \frac{\pi}{4}$ | L: $\frac{\sqrt{3}}{2}$ |
| 9. $3 \tan \frac{3\pi}{4} + \sin \frac{2\pi}{3}$ | E: $\frac{-1 + \sqrt{2}}{2}$ |
| 10. $\sin \frac{\pi}{3} + \csc \frac{5\pi}{3}$ | K: $\frac{-6 + \sqrt{3}}{2}$ |
| 11. $\left(\cos \frac{\pi}{2}\right)\left(\sec \frac{11\pi}{6}\right)$ | I: $\frac{3 + \sqrt{3}}{3}$ |
| | R: $\sqrt{3}$ |

10 8 7 4 1 2 6 11 3 5 9

"Why won't we ever run out of math teachers?"

Find the exact value for the following. Rationalize any radical denominator.
The answer to each problem will match a letter that will allow you to figure out the joke.

1. $\sin 30^\circ + \cos 60^\circ$

P: $\sqrt{3}$

2. $\cos 45^\circ - \cos 225^\circ$

E: $\sqrt{2}$

3. $\tan 60^\circ + \sin 90^\circ$

U: $-\frac{3}{2}$

4. $(\sin 60^\circ)(\tan 120^\circ)$

Y: $\frac{-\sqrt{2}}{4}$

5. $(\cos 240^\circ)(\sin 330^\circ)$

S: 0

6. $\sin \frac{\pi}{3} + \cos \frac{\pi}{6}$

H: 1

7. $\tan \frac{3\pi}{4} + \sin \frac{5\pi}{6}$

I: $1 + \sqrt{3}$

8. $(\sin 225^\circ)(\cos 300^\circ)$

L: $\frac{1}{4}$

9. $\left(\cos -\frac{\pi}{6}\right)\left(\sin \frac{7\pi}{6}\right)$

R: 4

M: $\frac{-\sqrt{3}}{4}$

T: $-\frac{1}{2}$

7

1

2

8

9

4

5

7

3

6

5

8

No Joking Around Trigonometric Identities

Prove each identity.

1. $\frac{\cot\theta}{\cos\theta} = \csc\theta$

2. $\cot\theta + \tan\theta = \csc\theta \sec\theta$

3. $\sin\theta + \frac{\cot\theta}{\sec\theta} = \csc\theta$

4. $\frac{1 - \cos^2\theta}{\cos\theta} \cdot \csc\theta = \tan\theta$

5. $2 \csc\theta = \frac{\sin\theta}{1 + \cos\theta} + \frac{\sin\theta}{1 - \cos\theta}$

6. $\frac{\cot^2\theta - 1}{\cot^2\theta + 1} = 2\cos^2\theta - 1$

No Joking Around Trigonometric Identities

Prove each identity.

1. $\sin^2\theta(1 + \cot^2\theta) = 1$

2. $\sec\theta = \frac{\tan\theta}{\csc\theta} + \cos\theta$

3. $\frac{1 + \cos\theta}{1 - \cos\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$

4. $(\cot\theta + \tan\theta)^2 = \csc^2\theta + \sec^2\theta$

5. $\frac{\cos\theta}{1 - \sin\theta} = \frac{1}{\sec\theta - \tan\theta}$

No Joking Around Trigonometric Identities Sum and Difference Formulas

Prove each identity.

1. $\frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

2. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$

3. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

4. $\frac{\cos(\alpha - \beta)}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta$

5. $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$

No Joking Around Trigonometric Identities Sum and Difference Formulas

Prove each identity.

1. $\sin(\pi + \Theta) = -\sin\Theta$

2. $\frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

3. $\cos\alpha\cos\beta(\tan\alpha + \tan\beta) = \sin(\alpha + \beta)$

4. $2\sin\alpha\cos\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$

No Joking Around Trigonometric Identities Double- and Half-Angle Formulas

Prove each identity.

1. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

2. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$

3. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

4. $\cot\theta + \tan\theta = 2\csc 2\theta$

No Joking Around Trigonometric Identities Double- and Half-Angle Formulas

Prove each identity.

1. $\cot\Theta = \frac{\sin 2\Theta}{2\sin^2\Theta}$

2. $\sin^2\Theta = \frac{\tan\Theta \sin 2\Theta}{2}$

3. $1 + \sin\Theta = \left(\sin\frac{\Theta}{2} + \cos\frac{\Theta}{2}\right)^2$

4. $\sin^2\Theta - 2\sin\Theta\cos\Theta + \cos^2\Theta = 1 - \sin 2\Theta$

5. $\frac{\sec^2\Theta}{2} + \tan\Theta = \frac{1 + \sin 2\Theta}{1 + \cos 2\Theta}$

No Joking Around Graphing Trigonometric Functions

Graph the following either by hand or with the aid of a graphing calculator. Compare all of the following graphs to $y = \sin x$ (if you are using a graphing calculator, then keep $y = \sin x$ as your first equation when comparing to the different problems.) Find the period, the amplitude, and the range for the following problems.

Graph the following, then find the period, the amplitude, and the range.

- | | |
|----------------------------|--|
| 1. $\sin(-x)$ | 1. period: _____
amplitude: _____
range: _____ |
| 2. $y = \sin(2x)$ | 2. period: _____
amplitude: _____
range: _____ |
| 3. $y = 2\sin x$ | 3. period: _____
amplitude: _____
range: _____ |
| 4. $y = \sin x + 2$ | 4. period: _____
amplitude: _____
range: _____ |
| 5. $y = 4\sin 4x$ | 5. period: _____
amplitude: _____
range: _____ |
| 6. $y = \frac{\sin x}{2}$ | 6. period: _____
amplitude: _____
range: _____ |
| 7. $y = \sin \frac{1}{2}x$ | 7. period: _____
amplitude: _____
range: _____ |

Think about what made the graph change. Write an equation for the situations below.

8. period = 2π
amplitude = 3
range = $0 \leq y \leq 6$

equations: _____

9. period = π
amplitude = $\frac{1}{2}$
range = $-2\frac{1}{2} \leq y \leq -1\frac{1}{2}$

equations: _____

Answers to the Joke Worksheets

- Joke 1: Seaman Says
- Joke 2: Dijon Vu
- Joke 4: UCLA
- Joke 5: Seen a Mall
- Joke 6: He's All Right Now
- Joke 7: Jab Well Done
- Joke 8: Mind Over Platter
- Joke 9: A Pair of Medics
- Joke 10: B-Negative
- Joke 11: To Erazona
- Joke 12: Crew Cuts
- Joke 13: He's Mist
- Joke 14: Corduroy Pillow
- Joke 15: Whipped Scream
- Joke 16: Inverse
- Joke 17: Gravity
- Joke 18: To Boo the Umpire
- Joke 19: Hex-aminations
- Joke 20: Siene
- Joke 21: Mugging
- Joke 22: Cell Phones
- Joke 23: Fission Chips
- Joke 24: A Dead End
- Joke 25: Aftermath
- Joke 26: A Nervous Wreck
- Joke 27: To Get a Boo-ster Shot
- Joke 28: Inca Blots
- Joke 29: He Did a So-So Job
- Joke 30: Fission Trip
- Joke 31: Go Out to Lunge
- Joke 32: Polaroids
- Joke 33: Psycho Path
- Joke 34: Freudian Slip
- Joke 35: In Case He Got a Hole in One
- Joke 37: Spoiled Milk
- Joke 38: They Multiply

Joke 39 (Answers can vary.)

1. $\frac{\cot\theta}{\cos\theta} = \csc\theta$

$$\cot\theta \cdot \frac{1}{\cos\theta} = \csc\theta$$

$$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

2. $\cot\theta + \tan\theta = \csc\theta \sec\theta$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \csc\theta \sec\theta$$

$$\frac{\cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} = \csc\theta \sec\theta$$

$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta \sin\theta} = \csc\theta \sec\theta$$

$$\frac{1}{\cos\theta \sin\theta} = \csc\theta \sec\theta$$

$$\sec\theta \csc\theta = \csc\theta \sec\theta$$

3. $\sin\theta + \frac{\cot\theta}{\sec\theta} = \csc\theta$

$$\sin\theta + \cot\theta \cdot \frac{1}{\sec\theta} = \csc\theta$$

$$\sin\theta + \frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{1} = \csc\theta$$

$$\sin\theta + \frac{\cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin\theta}{\sin\theta} \cdot \sin\theta + \frac{\cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \csc\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$\csc\theta + \csc\theta$$

$$4. \frac{1 - \cos^2 \Theta}{\cos \Theta} \cdot \csc \Theta = \tan \Theta$$

$$\frac{\sin^2 \Theta}{\cos \Theta} \cdot \frac{1}{\sin \Theta} = \tan \Theta$$

$$\frac{\sin \Theta}{\cos \Theta} = \tan \Theta$$

$$\tan \Theta = \tan \Theta$$

$$5. 2\csc \Theta = \frac{\sin \Theta}{1 + \cos \Theta} + \frac{\sin \Theta}{1 - \cos \Theta}$$

$$2\csc \Theta = \frac{1 + \cos \Theta}{1 - \cos \Theta} \cdot \frac{\sin \Theta}{1 + \cos \Theta} + \frac{\sin \Theta}{1 - \cos \Theta} + \frac{1 + \cos \Theta}{1 + \cos \Theta}$$

$$2\csc \Theta = \frac{\sin \Theta - \cos \Theta \sin \Theta + \sin \Theta + \cos \Theta \sin \Theta}{(1 - \cos \Theta)(1 + \cos \Theta)}$$

$$2\csc \Theta = \frac{2\sin \Theta}{1 - \cos \Theta + \cos \Theta - \cos^2 \Theta}$$

$$2\csc \Theta = \frac{2\sin \Theta}{1 - \cos^2 \Theta}$$

$$2\csc \Theta = \frac{2\sin \Theta}{\sin^2 \Theta}$$

$$2\csc \Theta = \frac{2}{\sin \Theta}$$

$$2\csc \Theta = 2\csc \Theta$$

$$6. \frac{\cot^2 \Theta - 1}{\cot^2 \Theta + 1} = 2\cos^2 \Theta - 1$$

$$\frac{\cot^2 \Theta - 1}{\csc^2 \Theta} = 2\cos^2 \Theta - 1$$

$$\frac{\cot^2 \Theta}{\csc^2 \Theta} - \frac{1}{\csc^2 \Theta} = 2\cos^2 \Theta - 1$$

$$\cot^2 \Theta \cdot \frac{1}{\csc^2 \Theta} - \sin^2 \Theta = 2\cos^2 \Theta - 1$$

$$\frac{\cos^2 \Theta}{\sin^2 \Theta} \cdot \sin^2 \Theta - \sin^2 \Theta = 2\cos^2 \Theta - 1$$

$$\cos^2 \Theta - \sin^2 \Theta = 2\cos^2 \Theta - 1$$

$$\cos^2 \Theta - (1 - \cos^2 \Theta) = 2\cos^2 \Theta - 1$$

$$\cos^2 \Theta - 1 + \cos^2 \Theta = 2\cos^2 \Theta - 1$$

$$2\cos^2 \Theta - 1 = 2\cos^2 \Theta - 1$$

Joke 40 (Answers can vary.)

1. $\sin^2\Theta (1 + \cot^2\Theta) = 1$

$$\sin^2\Theta (\csc^2\Theta) = 1$$

$$\sin^2\Theta \left(\frac{1}{\sin^2\Theta} \right) = 1$$

$$1 = 1$$

2. $\sec\Theta = \frac{\tan\Theta}{\csc\Theta} + \cos\Theta$

$$\sec\Theta = \tan\Theta \cdot \frac{1}{\csc\Theta} + \cos\Theta$$

$$\sec\Theta = \frac{\sin\Theta}{\cos\Theta} \cdot \sin\Theta + \cos\Theta$$

$$\sec\Theta = \frac{\sin^2\Theta}{\cos\Theta} + \cos\Theta \cdot \frac{\cos\Theta}{\cos\Theta}$$

$$\sec\Theta = \frac{\sin^2\Theta + \cos^2\Theta}{\cos\Theta}$$

$$\sec\Theta = \frac{1}{\cos\Theta}$$

$$\sec\Theta = \sec\Theta$$

3. $\frac{1 + \cos\Theta}{1 - \cos\Theta} = \frac{\sec\Theta + 1}{\sec\Theta - 1}$

$$\frac{1 + \cos\Theta}{1 - \cos\Theta} = \frac{\frac{1}{\cos\Theta} + 1}{\frac{1}{\cos\Theta} - 1}$$

$$\frac{1 + \cos\Theta}{1 - \cos\Theta} = \frac{\frac{1 + \cos\Theta}{\cos\Theta}}{\frac{1 - \cos\Theta}{\cos\Theta}}$$

$$\frac{1 + \cos\Theta}{1 - \cos\Theta} = \frac{1 + \cos\Theta}{1 - \cos\Theta}$$

4. $(\cot\Theta + \tan\Theta)^2 = \csc^2\Theta + \sec^2\Theta$

$$(\cot\Theta + \tan\Theta)(\cot\Theta + \tan\Theta) = \csc^2\Theta + \sec^2\Theta$$

$$\cot^2\Theta + 2\cot\Theta\tan\Theta + \tan^2\Theta = \csc^2\Theta + \sec^2\Theta$$

$$\cot^2\Theta + 2\left(\frac{\cos\Theta}{\sin\Theta}\right)\left(\frac{\sin\Theta}{\cos\Theta}\right) + \tan^2\Theta = \csc^2\Theta + \sec^2\Theta$$

$$\cot^2\Theta + 2 + \tan^2\Theta = \csc^2\Theta + \sec^2\Theta$$

$$(\cot^2\Theta + 1) + (1 + \tan^2\Theta) = \csc^2\Theta + \sec^2\Theta$$

$$\csc^2\Theta + \sec^2\Theta = \csc^2\Theta + \sec^2\Theta$$

$$5. \frac{\cos\Theta}{1 - \sin\Theta} = \frac{1}{\sec\Theta - \tan\Theta}$$

$$\frac{\cos\Theta}{1 - \sin\Theta} = \frac{1}{\frac{1}{\cos\Theta} - \frac{\sin\Theta}{\cos\Theta}}$$

$$\frac{\cos\Theta}{1 - \sin\Theta} = \frac{1}{\frac{1 - \sin\Theta}{\cos\Theta}}$$

$$\frac{\cos\Theta}{1 - \sin\Theta} = \frac{\cos\Theta}{1 - \sin\Theta}$$

Joke 41 (Answers can vary.)

$$1. \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \tan\alpha + \tan\beta$$

$$\tan\alpha + \tan\beta = \tan\alpha + \tan\beta$$

$$2. \sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$$

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta = 2\sin\alpha\cos\beta$$

$$2\sin\alpha\cos\beta = 2\sin\alpha\cos\beta$$

$$3. \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$$

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta = 2\cos\alpha\cos\beta$$

$$2\cos\alpha\cos\beta = 2\cos\alpha\cos\beta$$

$$4. \frac{\cos(\alpha - \beta)}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta$$

$$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta$$

$$\frac{\cos\alpha\cos\beta}{\cos\alpha\sin\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta$$

$$\frac{\cos\beta}{\sin\beta} + \frac{\sin\alpha}{\cos\alpha} = \tan\alpha + \cot\beta$$

$$\cot\beta + \tan\alpha = \tan\alpha + \cot\beta$$

$$5. \sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta$$

$$(\sin\alpha\cos\beta + \cos\alpha\sin\beta)(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = \sin^2\alpha - \sin^2\beta$$

$$\sin^2\alpha\cos^2\beta + \sin\alpha\cos\alpha\sin\beta\cos\beta - \sin\alpha\cos\alpha\sin\beta\cos\beta - \cos^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$$

$$\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$$

$$\sin^2\alpha(1 - \sin^2\beta) - (1 - \sin^2\alpha)\sin^2\beta = \sin^2\alpha - \sin^2\beta$$

$$\sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$$

$$\sin^2\alpha - \sin^2\beta = \sin^2\alpha - \sin^2\beta$$

Joke 42 (Answers can vary.)

$$1. \sin(\pi + \Theta) = -\sin\Theta$$

$$\sin\pi\cos\Theta + \cos\pi\sin\Theta = -\sin\Theta$$

$$(0)(\cos\Theta) + (-1)(\sin\Theta) = -\sin\Theta$$

$$-\sin\Theta = -\sin\Theta$$

$$2. \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\alpha} = \tan\alpha + \tan\beta$$

$$\tan\alpha + \tan\beta = \tan\alpha + \tan\beta$$

$$3. \cos\alpha\cos\beta(\tan\alpha + \tan\beta) = \sin(\alpha + \beta)$$

$$\cos\alpha\cos\beta\tan\alpha + \cos\alpha\cos\beta\tan\beta = \sin(\alpha + \beta)$$

$$\cos\alpha\cos\beta \cdot \frac{\sin\alpha}{\cos\alpha} + \cos\alpha\cos\beta \cdot \frac{\sin\beta}{\cos\alpha} = \sin(\alpha + \beta)$$

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha + \beta)$$

$$\sin(\alpha + \beta) = \sin(\alpha + \beta)$$

$$4. 2\sin\alpha\cos\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

$$2\sin\alpha\cos\beta = \sin\alpha\cos\beta - \cos\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$2\sin\alpha\cos\beta = 2\sin\alpha\cos\beta$$

Joke 43 (Answers can vary.)

1. $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta) = 1 + \sin 2\theta$$

$$\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta = 1 + \sin 2\theta$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + \sin 2\theta$$

$$1 + 2\sin\theta\cos\theta = 1 + \sin 2\theta$$

$$1 + \sin 2\theta = 1 + \sin 2\theta$$

2. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$

$$\frac{2\sin\theta\cos\theta}{1 + \cos^2\theta - \sin^2\theta} = \tan\theta$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta + (1 - \sin^2\theta)} = \tan\theta$$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta + \cos^2\theta} = \tan\theta$$

$$\frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \tan\theta$$

3. $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \cos 2\theta$$

$$(\cos^2\theta - \sin^2\theta)(1) = \cos 2\theta$$

$$\cos 2\theta = \cos 2\theta$$

4. $\cot\theta + \tan\theta = 2\csc 2\theta$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin 2\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

Joke 44 (Answers can vary.)

$$1. \cot \Theta = \frac{\sin 2\Theta}{2\sin^2 \Theta}$$

$$\cot \Theta = \frac{2\sin \Theta \cos \Theta}{2\sin^2 \Theta}$$

$$\cot \Theta = \frac{\cos \Theta}{\sin \Theta}$$

$$\cot \Theta = \cot \Theta$$

$$2. \sin^2 \Theta = \frac{\tan \Theta \sin 2\Theta}{2}$$

$$\sin^2 \Theta = \frac{\tan \Theta (2\sin \Theta \cos \Theta)}{2}$$

$$\sin^2 \Theta = \tan \Theta \sin \Theta \cos \Theta$$

$$\sin^2 \Theta = \frac{\sin \Theta}{\cos \Theta} \cdot \sin \Theta \cdot \cos \Theta$$

$$\sin^2 \Theta = \sin^2 \Theta$$

$$3. 1 + \sin \Theta = \sin\left(\frac{\Theta}{2}\right) + \cos\left(\frac{\Theta}{2}\right)^2$$

$$1 + \sin \Theta = \left(\sin \frac{\Theta}{2} + \cos \frac{\Theta}{2}\right)\left(\sin \frac{\Theta}{2} + \cos \frac{\Theta}{2}\right)$$

$$1 + \sin \Theta = \left(\sin \frac{\Theta}{2}\right)\left(\sin \frac{\Theta}{2}\right) + 2\left(\sin \frac{\Theta}{2}\right)\left(\cos \frac{\Theta}{2}\right) + \left(\cos \frac{\Theta}{2}\right)\left(\cos \frac{\Theta}{2}\right)$$

$$1 + \sin \Theta = \left(\sqrt{\frac{1 - \cos \Theta}{2}}\right)\left(\sqrt{\frac{1 - \cos \Theta}{2}}\right) + 2\left(\sqrt{\frac{1 - \cos \Theta}{2}}\right)\left(\sqrt{\frac{1 + \cos \Theta}{2}}\right) + \left(\sqrt{\frac{1 + \cos \Theta}{2}}\right)\left(\sqrt{\frac{1 + \cos \Theta}{2}}\right)$$

$$1 + \sin \Theta = \frac{1 - \cos \Theta}{2} + 2\sqrt{\frac{1 - \cos^2 \Theta}{4}} + \frac{1 + \cos^2 \Theta}{2}$$

$$1 + \sin \Theta = \frac{1}{2} - \frac{\cos \Theta}{2} + 2 \cdot \frac{\sqrt{\sin^2 \Theta}}{2} + \frac{1}{2} + \frac{\cos \Theta}{2}$$

$$1 + \sin \Theta = 1 + \sin \Theta$$

$$4. \sin^2 \Theta - 2\sin \Theta \cos \Theta + \cos^2 \Theta = 1 - \sin 2\Theta$$

$$(\sin^2 \Theta + \cos^2 \Theta - 2\sin \Theta \cos \Theta = 1 - \sin 2\Theta)$$

$$1 - \sin 2\Theta = 1 - \sin 2\Theta$$

$$5. \frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{1 + \sin 2\Theta}{1 + \cos 2\Theta}$$

$$\frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{1 + 2\sin \Theta \cos \Theta}{1 + 2\cos^2 \Theta - 1}$$

$$\frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{1 + 2\sin \Theta \cos \Theta}{2\cos^2 \Theta}$$

$$\frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{1}{2\cos^2 \Theta} + \frac{2\sin \Theta \cos \Theta}{2\cos^2 \Theta}$$

$$\frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{\sec^2 \Theta}{2} + \frac{\sin \Theta}{\cos \Theta}$$

$$\frac{\sec^2 \Theta}{2} + \tan \Theta = \frac{\sec^2 \Theta}{2} + \tan \Theta$$

Joke 45

1. period = 2π
amplitude = 1
range = $-1 \leq y \leq 1$

2. period = π
amplitude = 1
range = $-1 \leq y \leq 1$

3. period = 2π
amplitude = 2
range = $-2 \leq y \leq 2$

4. period = 2π
amplitude = 1
range = $1 \leq y \leq 3$

5. period = $\frac{\pi}{2}$
amplitude = 4
range = $-4 \leq y \leq 4$

6. period = 2π
amplitude = $\frac{1}{2}$
range = $-\frac{1}{2} \leq y \leq \frac{1}{2}$

7. period = 4π
amplitude = 1
range = $-1 \leq y \leq 1$

8. $y = 3 \sin x + 3$

9. $y = \frac{1}{2} \sin 2x - 2$