Level 2: Volume 2-Atomic Clocks Are Ticking

1.1 **Identify the rate of growth or decay represented by each of the following exponential equations:**

a. $y = 229 \bullet 0.44^x$ **b.** $y = 750 \bullet 1.10^x$

c. $y = 99.95 \bullet 3^x$ **d.** $y = 82 \bullet 0.7^x$

a. This represents decay and the rate of decay is 100 – 44 = -56%
b. This represents growth because 100% + 10 % = 110% So the rate of growth is 10%
c. This also represents growth 100% + 200% = 300% so the rate of growth is 200%
d. This represents decay. The rate of decay is 100% - 70% = 30%.

1.2 **Evaluate each equation in Problem 1.1 for the following values:** $x = 3$, $x = 0.5$, **and** $x = 1/3$.

$y = 229 \bullet 0.44^x$	$y = 750 \bullet 1.10^x$	$y = 99.95 \bullet 3^x$	$y = 82 \bullet 0.7^x$
$y = 229 \bullet 0.44^3 =$ 19.507	$y = 750 \bullet 1.10^3 =$ 998.25	$y = 99.95 \bullet 3^3 =$ 2698.7	$y = 82 \bullet 0.7^3 =$ 28.126
$y = 229 \bullet 0.44^{0.5} =$ 151.9	$y = 750 \bullet 1.10^{0.5} =$ 786.61	$y = 99.95 \bullet 3^{0.5} =$ 173.12	$y = 82 \bullet 0.7^{0.5} =$ 68.606
$y = 229 \bullet 0.44^{\frac{1}{3}} =$ 174.18	$y = 750 \bullet 1.10^{\frac{1}{3}} =$ 774.21	$y = 99.95 \bullet 3^{\frac{1}{3}} =$144.15	$y = 82 \bullet 0.7^{\frac{1}{3}} =$ 72.808

1.3 Evaluate each of the expressions below.

a. $64^{1/2}$ **b.** 16^4

c. $15625^{1/6}$ **d.** $\sqrt[3]{343}$

a. $\sqrt{64} = 8$
b. 16*16*16*16 = 65536
c, $\sqrt[6]{15625} = 5$
d. 7

1.4 Write an exponential equation in the form $y = a \bullet b^x$ to model each of the following situations:

a. an initial population of 63 and a growth rate of 14%

b. a decay rate of 0.8% and an initial population of 741

c. an initial population of 2 and a growth rate of 250%

d. an initial population of $4 \bullet 10^9$ and a decay rate of 25%

a. $y = 63 \bullet 1.14^x$
b. $y = 741 \bullet 0.992^x$
c. $y = 2 \bullet 3.5^x$
d. $y = (4 \bullet 10^9) \bullet 0.75^x$

1.5 From the moment a new car is driven off the lot, its value begins to depreciate. This situation can be modeled using exponential decay. The table below shows how the mean value of a car decreases over time.

Year after Purchase	Mean Value ($)
0	26,756
1	19,700
2	16,738
3	13,937
4	11,775
5	9750

a. Determine the rate of decay in this car's value.

b. Write an equation in the form $y = a \bullet b^x$ to model this situation.

c. Use your equation to estimate the value of the car 10 years after it was purchased.

These are the decay rates between each year:

-26%
-15%
-17%
-16%
-17%

The average of these rates is
-18%

So a loss of 18% of its value each year.

b. The equation is : $y = 26756 \bullet 0.82^x$

c. After 0 years the car will be worth:

$y = 26756 \bullet 0.82^{10}$ = $3,678

Additional Review—for use with Activity 2

2.1 Evaluate each of the following expressions:

a. 7^{-2} **b.** $81^{-1/4}$ **c.** $\frac{1}{6^{-3}}$

a. $\frac{1}{7^2} = \frac{1}{49}$

b. $\frac{1}{\sqrt[4]{81}} = \frac{1}{3}$

c. $6^3 = 216$

2.2 Rewrite each expression below using positive exponents.

a. $x^{-1/2}$ **b.** $x^{-2/3}$ **c.** $\sqrt[7]{x}$

d. $\frac{1}{x^{-9}}$ **e.** $\frac{1}{\sqrt[5]{32}}$ **f.** $\frac{1}{x^{-3/5}}$

a. $\frac{1}{x^{\frac{1}{2}}}$

b. $\frac{1}{x^{\frac{2}{3}}}$ or $\left(\frac{1}{x^2}\right)^{\frac{1}{3}}$ or $\left(\frac{1}{x^{\frac{1}{3}}}\right)^2$

c. $x^{\frac{1}{7}}$

d. x^9

e. $\frac{1}{32^{\frac{1}{5}}}$

f. $x^{\frac{3}{5}}$

2.3 Simplify each of the following expressions:

a. $(x^5)^2$ **b.** $(x^4)^{-6}$ **c.** $(x^9)^{1/3}$

a. x^{10}

b. x^{-24} or even better $\frac{1}{x^{24}}$

c. x^3

2.4 **a.** Rewrite $(1/5)^4$ using an integer as the base

b. Rewrite $(1/x)^y$ using x as the base.

c. Rewrite $(2/7)^5$ without using parentheses.

d. Rewrite $(2/7)^5$ using a negative exponent.

a. 5^{-4}

b. x^{-y}

c. $\frac{2^5}{7^5}$ or $\frac{32}{16807}$

d. $\left(\frac{7}{2}\right)^{-5}$

2.5 Solve each of the following equations for x.

a. $x^8 = 390625$

b. x ` $= 7$

c. $x^{3/5} = 2.93$

d. $10x^4 = 12960$

a.
$$x^8 = 390625$$
$$\left(x^8\right)^{\frac{1}{8}} = (390625)^{\frac{1}{8}}$$
$$x = \sqrt[8]{390625} = 5$$

b.
$$x^{\frac{1}{3}} = 7$$
$$\left(x^{\frac{1}{3}}\right)^3 = 7^3$$
$$x = 343$$

c.
$$x^{\frac{3}{5}} = 2.93$$
$$\left(x^{\frac{3}{5}}\right)^{\frac{5}{3}} = 2.93^{\frac{5}{3}}$$
$$x = \sqrt[3]{2.93}^5$$
$$x = 1.4309^5$$
$$x \approx 6$$

d.

$$10x^4 = 12960$$

$$\frac{10x^4}{10} = \frac{12960}{10}$$

$$x^4 = 1296$$

$$\left(x^4\right)^{\frac{1}{4}} = 1296^{\frac{1}{4}}$$

$$x = \sqrt[4]{1296}$$

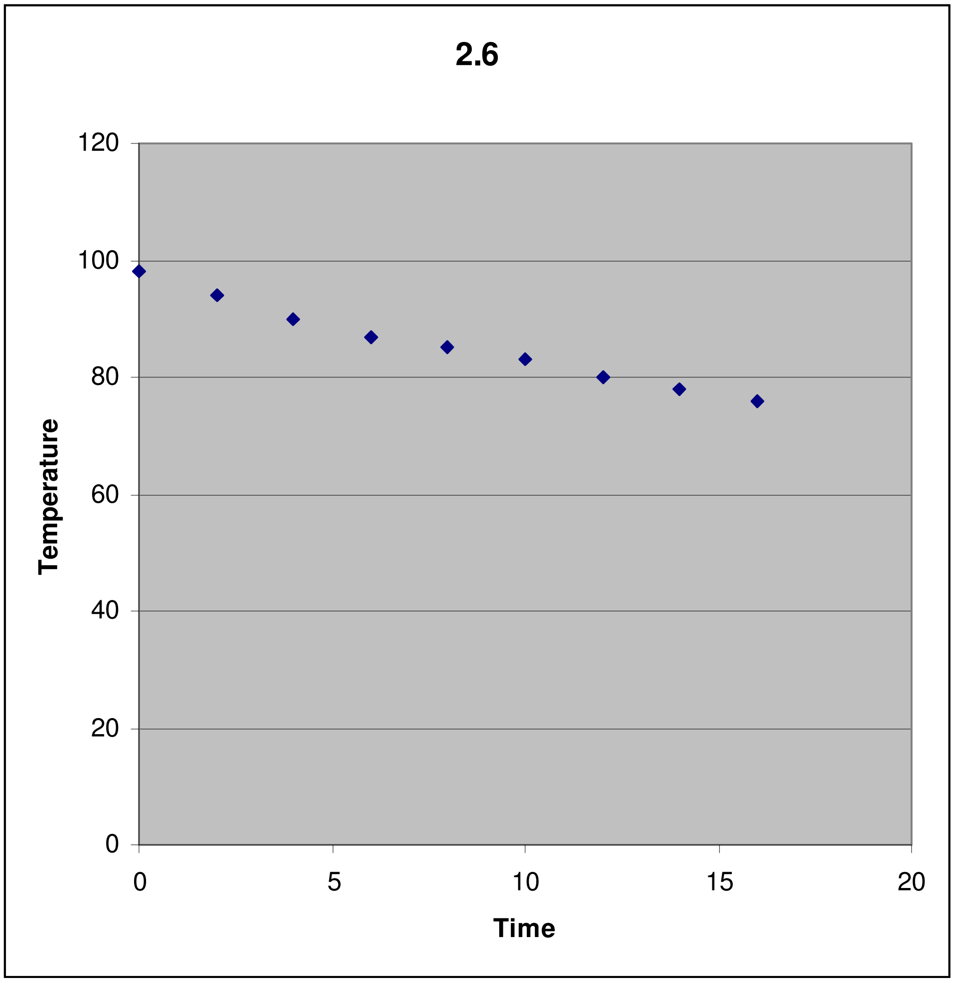
$$x = 6$$

2.6 This table shows the change in the temperature of a cup of coffee over time.

Time (min)	Temperature (°C)
0	98
2	94
4	90
6	87
8	85
10	83
12	80
14	78
16	76

a. Create a scatterplot of the data.

b. Find the mean percent decrease in temperature over a 2-min interval, then write an equation of the form $y = a \bullet b^x$ that models the data.



a

b. The mean rate of decrease is –3% so the equation for the model would be: $y = 98 \bullet 0.97^x$