HW Review 1/3/2012

Applying the Concepts 6–4

Smart People

Assume you are thinking about starting a Mensa chapter in your home town of Visiala, California, which has a population of about 10,000 people. You need to know how many people would qualify for Mensa, which requires an IQ of at least 130. You realize that IQ is normally distributed with a mean of 100 and a standard deviation of 15. Complete the following.

1. Find the approximate number of people in Visiala that are eligible for Mensa.
2. Is it reasonable to continue your quest for a Mensa chapter in Visiala?
3. How would you proceed to find out how many of the eligible people would actually join the new chapter? Be specific about your methods of gathering data.
4. What would be the minimum IQ score needed if you wanted to start an Ultra-Mensa club that included only the top 1% of IQ scores?

See page 344 for the answers.

Exercises 6–4

1. The average admission charge for a movie is \$5.39. If the distribution of admission charges is normal with a standard deviation of \$0.79, what is the probability that a randomly selected admission charge is less than \$3.00?

Source: *N.Y. Times Almanac.* 0.0012

2. The average salary for first-year teachers is \$27,989. If the distribution is approximately normal with $\sigma = \$3250$, what is the probability that a randomly selected first-year teacher makes these salaries?

a. Between \$20,000 and \$30,000 a year

b. Less than \$20,000 a year

Source: *N.Y. Times Almanac.*

3. The average daily jail population in the United States is 618,319. If the distribution is normal and the standard deviation is 50,200, find the probability that on a randomly selected day the jail population is

a. Greater than 700,000. 0.0516

b. Between 500,000 and 600,000. 0.3503

Source: *N.Y. Times Almanac.*

4. The national average SAT score is 1019. If we assume a normal distribution with $\sigma = 90$, what is the 90th percentile score? What is the probability that a randomly selected score exceeds 1200?

Source: *N.Y. Times Almanac.*

5. The average number of calories in a 1.5-ounce chocolate bar is 225. Suppose that the distribution of calories is approximately normal with $\sigma = 10$. Find the probability that a randomly selected chocolate bar will have

a. Between 200 and 220 calories. 0.3023

b. Less than 200 calories. 0.0062

Source: *The Doctor's Pocket Calorie, Fat, and Carbohydrate Counter.*

6. The average age of CEOs is 56 years. Assume the variable is normally distributed. If the standard deviation is 4 years, find the probability that the age of a randomly selected CEO will be in the following range.

a. Between 53 and 59 years old

b. Between 58 and 63 years old

c. Between 50 and 55 years old

Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

7. The average salary for a Queens College full professor is \$85,900. If the average salaries are normally distributed with a standard deviation of \$11,000, find these probabilities.

a. The professor makes more than \$90,000. 0.3557

b. The professor makes more than \$75,000. 0.8389

Source: AAUP, *Chronicle of Higher Education.*

8. Full-time Ph.D. students receive an average of \$12,837 per year. If the average salaries are normally distributed with a standard deviation of \$1500, find these probabilities.

a. The student makes more than \$15,000.

b. The student makes between \$13,000 and \$14,000.

Source: U.S. Education Dept., *Chronicle of Higher Education.*

9. A survey found that people keep their microwave ovens an average of 3.2 years. The standard deviation is 0.56 year. If a person decides to buy a new microwave oven, find the probability that he or she has owned the old oven for the following amount of time. Assume the variable is normally distributed.

a. Less than 1.5 years 0.0012

b. Between 2 and 3 years 0.3432

c. More than 3.2 years 0.500

d. What percent of microwave ovens would be replaced if a warranty of 18 months were given? About 0.12% of the microwave ovens would be replaced.

10. The average commute to work (one way) is 25.5 minutes according to the 2000 Census. If we assume that commuting times are normally distributed with a standard deviation of 6.1 minutes, what is the probability that a randomly selected commuter spends more than 30 minutes a day commuting one way?
Source: *N.Y. Times Almanac.*

11. The average credit card debt for college seniors is $3262. If the debt is normally distributed with a standard deviation of $1100, find these probabilities.

a. That the senior owes at least $1000 0.9803

b. That the senior owes more than $4000 0.2514

c. That the senior owes between $3000 and $4000 0.3434
Source: *USA TODAY.*

12. The average time a person spends at the Barefoot Landing Seaquarium is 96 minutes. The standard deviation is 17 minutes. Assume the variable is normally distributed. If a visitor is selected at random, find the probability that he or she will spend the following time at the seaquarium.

a. At least 120 minutes

b. At most 80 minutes

c. Suggest a time for a bus to return to pick up a group of tourists. The bus should make a trip every 130 minutes or $\mu + 2\sigma$.

13. The average time for a mail carrier to cover his route is 380 minutes, and the standard deviation is 16 minutes. If one of these trips is selected at random, find the probability that the carrier will have the following route time. Assume the variable is normally distributed.

a. At least 350 minutes 0.9699

b. At most 395 minutes 0.8264

c. How might a mail carrier estimate a range for the time he or she will spend en route? Use the range rule of thumb. The range is about $4 \times 16 = 64$ minutes.

14. During October, the average temperature of Whitman Lake is 53.2° and the standard deviation is 2.3°. Assume the variable is normally distributed. For a randomly selected day in October, find the probability that the temperature will be as follows.

a. Above 54°

b. Below 60°

c. Between 49 and 55°

d. If the lake temperature were above 60°, would you call it very warm?

15. The average waiting time to be seated for dinner at a popular restaurant is 23.5 minutes, with a standard deviation of 3.6 minutes. Assume the variable is normally distributed. When a patron arrives at the restaurant for dinner, find the probability that the patron will have to wait the following time.

a. Between 15 and 22 minutes 0.3281

b. Less than 18 minutes or more than 25 minutes 0.4002

c. Is it likely that a person will be seated in less than 15 minutes? Not usually

16. A local medical research association proposes to sponsor a footrace. The average time it takes to run the course is 45.8 minutes with a standard deviation of 3.6 minutes. If the association decides to include only the top 25% of the racers, what should be the cutoff time in the tryout run? Assume the variable is normally distributed. Would a person who runs the course in 40 minutes qualify?

17. A marine sales dealer finds that the average price of a previously owned boat is $6492. He decides to sell boats that will appeal to the middle 66% of the market in terms of price. Find the maximum and minimum prices of the boats the dealer will sell. The standard deviation is $1025, and the variable is normally distributed. Would a boat priced at $5550 be sold in this store?
$5518.25 $< \mu <$ $7465.75; yes

18. The average charitable contribution itemized per income tax return in Pennsylvania is $792. Suppose that the distribution of contributions is normal with a standard deviation of $103. Find the limits for the middle 50% of contributions.
Source: IRS, *Statistics of Income Bulletin.*

19. A contractor decided to build homes that will include the middle 80% of the market. If the average size of homes built is 1810 square feet, find the maximum and minimum sizes of the homes the contractor should build. Assume that the standard deviation is 92 square feet and the variable is normally distributed. The maximum size is 1927.76 square feet; the minimum size is 1692.24 square feet.
Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

20. If the average price of a new home is $145,500, find the maximum and minimum prices of the houses that a contractor will build to include the middle 80% of the market. Assume that the standard deviation of prices is $1500 and the variable is normally distributed.
Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

21. The average price of a personal computer (PC) is $949. If the computer prices are approximately normally distributed and $\sigma =$ $100, what is the probability that a randomly selected PC costs more than $1200? The least expensive 10% of personal computers cost less than what amount? 0.006; $821
Source: *N.Y. Times Almanac.*

22. To help students improve their reading, a school district decides to implement a reading program. It is to be administered to the bottom 5% of the students in the district, based on the scores on a reading achievement exam. If the average score for the students in the district is 122.6, find the cutoff score that will make a student eligible for the program. The standard deviation is 18. Assume the variable is normally distributed.

23. An automobile dealer finds that the average price of a previously owned vehicle is $8256. He decides to sell cars that will appeal to the middle 60% of the market in terms of price. Find the maximum and minimum prices of the cars the dealer will sell. The standard deviation is $1150, and the variable is normally distributed.
The maximum price is $9222, and the minimum price is $7290.

24. The average age of Amtrak passenger train cars is 19.4 years. If the distribution of ages is normal and 20% of the cars are older than 22.8 years, find the standard deviation.

Source: *N.Y. Times Almanac.*

25. The average length of a hospital stay is 5.9 days. If we assume a normal distribution and a standard deviation of 1.7 days, 15% of hospital stays are less than how many days? Twenty-five percent of hospital stays are longer than how many days? 4.13; 7.04

Source: *N.Y. Times Almanac.*

26. A mandatory competency test for high school sophomores has a normal distribution with a mean of 400 and a standard deviation of 100.
   a. The top 3% of students receive $500. What is the minimum score you would need to receive this award?
   b. The bottom 1.5% of students must go to summer school. What is the minimum score you would need to stay out of this group?

27. An advertising company plans to market a product to low-income families. A study states that for a particular area, the average income per family is $24,596 and the standard deviation is $6256. If the company plans to target the bottom 18% of the families based on income, find the cutoff income. Assume the variable is normally distributed. $18,840.48

28. If a one-person household spends an average of $40 per week on groceries, find the maximum and minimum dollar amounts spent per week for the middle 50% of one-person households. Assume that the standard deviation is $5 and the variable is normally distributed.

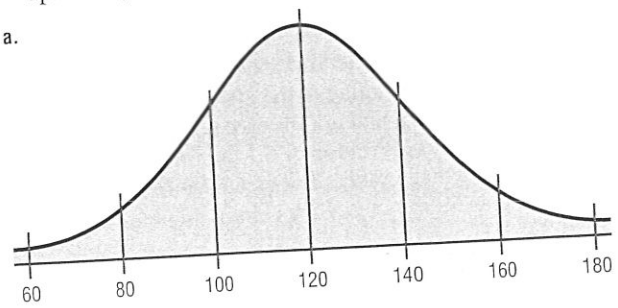
Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

29. The mean lifetime of a wristwatch is 25 months, with a standard deviation of 5 months. If the distribution is normal, for how many months should a guarantee be made if the manufacturer does not want to exchange more than 10% of the watches? Assume the variable is normally distributed. 18.6 months

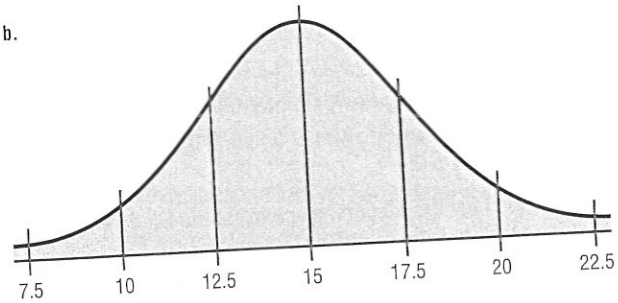
30. To qualify for security officers' training, recruits are tested for stress tolerance. The scores are normally distributed, with a mean of 62 and a standard deviation of 8. If only the top 15% of recruits are selected, find the cutoff score.

31. In the distributions shown, state the mean and standard deviation for each. *Hint:* See Figures 6–5 and 6–6. Also the vertical lines are 1 standard deviation apart. *a.* $\mu = 120, \sigma = 20$ *b.* $\mu = 15, \sigma = 2.5$ *c.* $\mu = 30, \sigma = 5$

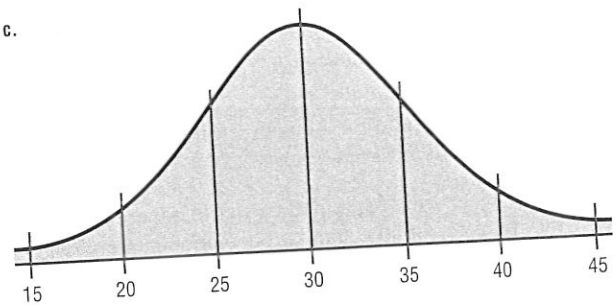
a.



b.



c.



32. Suppose that the mathematics SAT scores for high school seniors for a specific year have a mean of 456 and a standard deviation of 100 and are approximately normally distributed. If a subgroup of these high school seniors, those who are in the National Honor Society, is selected, would you expect the distribution of scores to have the same mean and standard deviation? Explain your answer.

33. Given a data set, how could you decide if the distribution of the data was approximately normal?
There are several mathematics tests that can be used.

34. If a distribution of raw scores were plotted and then the scores were transformed to z scores, would the shape of the distribution change? Explain your answer.

35. In a normal distribution, find σ when $\mu = 110$ and 2.87% of the area lies to the right of 112. 1.05

36. In a normal distribution, find μ when σ is 6 and 3.75% of the area lies to the left of 85.

37. In a certain normal distribution, 1.25% of the area lies to the left of 42, and 1.25% of the area lies to the right of 48. Find μ and σ. $\mu = 45, \sigma = 1.34$