AP Calculus AB 2018-2019 Summer Assignment
June 2018

Dear Future AP Calculus Student,

I hope you are excited for the year of Calculus that we will be pursuing together. I don't know how much you know about Calculus, but it is not like any other branch of mathematics that you have learned so far in your math careers. We will be having a lot of fun and doing a lot of work, learning about limits and derivatives (for the first semester) and integrals (for the second semester). You don't need to know what those things are (yet) but I will tell you that Calculus is described as the "mathematics of change", how fast things change, how to predict change, and how to use information about change to understand the systems themselves.

Actually, in some ways, Calculus is taking what you already know a step further. You know how to find the slope of a line, right? You probably don't know how to find the slope of a curve because the slope is constantly changing. Calculus helps us accomplish this. So traditional math tells us how to find the length of a rope pulled taut and Calculus tells us how to find the length of a rope with slack (a curved rope) "Traditional" math tells us how to find the area of a flat rectangular roof but Calculus tells us how to find the area of a curved dome-shaped roof. Get the idea?

In order to give you a head start in understanding Calculus, I have designed a few things for you to do. I've tried to make it fairly straightforward so you can still enjoy the summer, but I think this will get us rolling more quickly in August and therefore will put you in a better position to take your AP exam in May. Please answer the questions that follow and bring this packet with you for the first day of school; Wednesday August 29th, 2018. Feel free to send me an email if you have any questions.

See you in August,

Mr Lemay
Calculus AB Teacher
http://danlemay.net (Links for the course and help)
dlemay@oxbowhs.org

AP Calculus AB
2018-2019
Mr Lemay

Summer Assignment
Oxbow High School
AP Calculus AB Summer Review Packet

Name____________

Name______________________

Read the following directions carefully!

1. Before answering any questions, read through the given notes and examples for each topic.
2. If you want some more self help, see danlemay.net>Calculus for directions in accessing Khan Academy help specific for this packet.
3. This packet and your work is to be handed in on the first day of school Wednesday August 29th, 2018.
4. All work must be shown in the packet OR on separate paper attached to the packet.
5. Student Choice: You may pick 14 of the 17 sections to complete but you are responsible for knowing the contents of the entire packet.
6. Completion of this packet is worth 3 HW grades.
7. All work must be organized and neatly written.

Teacher: Write your class code (or email) in the form below.

See danlemay.net >Calculus For links To Topics.

KHANACADEMY

How to join your teacher's Khan Academy class

Sign in First.

1. Sign up at **khanacademy.org** (or log in if you already have an account).
2. Visit **khanacademy.org/coaches** (the "Coaches" tab in your profile).
3. In the "Add a coach" field, enter the class code.

   Class code: Y8QJUP

4. You're set. Now click **Home** to start learning!

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "f of g of x" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ __________
2. $g(-3) =$ __________
3. $f(t+1) =$ __________
4. $f[g(-2)] =$ __________
5. $g[f(m+2)] =$ __________
6. $[f(x)]^2 - 2g(x) =$ ________

Let $f(x) = \sin(2x)$ **Find each exactly.**

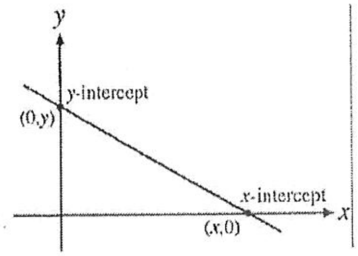
7. $f\left(\dfrac{\pi}{4}\right) =$ __________
8. $f\left(\dfrac{2\pi}{3}\right) =$ __________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h[f(-2)] =$ ________
10. $f[g(x-1)] =$ ________
11. $g\left[h(x^3)\right] =$ ________

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y=x^2-2x-3$, find all intercepts.

$\underline{x-\text{int. } (Let\ y=0)}$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts } (-1,0)\ and\ (3,0)$

$\underline{y-\text{int. } (Let\ x=0)}$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept } (0,-3)$

Find the x and y intercepts for each.

12. $y=2x-5$

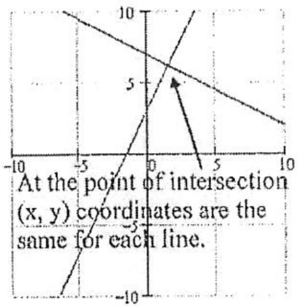
13. $y=x^2+x-2$

14. $y=x\sqrt{16-x^2}$

15. $y^2=x^3-4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2^{nd} Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{matrix} x^2 - y = 3 \\ x - y = 1 \end{matrix}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x=2$ and $x = -1$ to find y

Points of Intersection: (2,1) and (−1,−2)

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{matrix} x + y = 8 \\ 4x - y = 7 \end{matrix}$

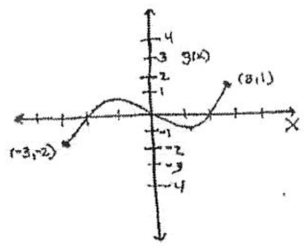
17. $\begin{matrix} x^2 + y = 6 \\ x + y = 4 \end{matrix}$

18. $\begin{matrix} x = 3 - y^2 \\ y = x - 1 \end{matrix}$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs ~~out~~ of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is $[-3,3]$, that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is $[-2,1]$, all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x)=x^2-5$

20. $f(x)=-\sqrt{x+3}$

21. $f(x)=3\sin x$

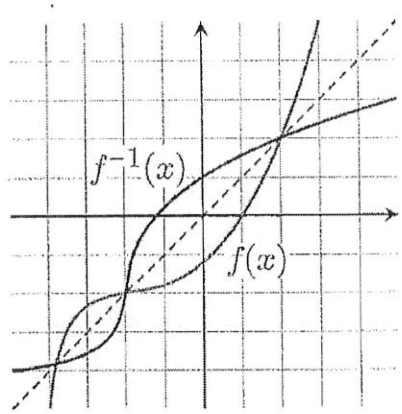
22. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x)=2x+1$

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

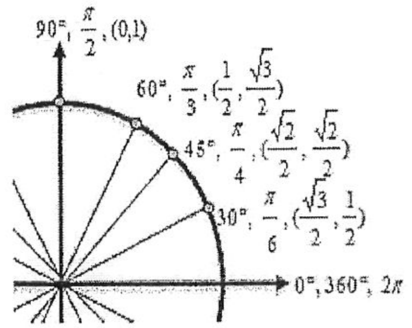
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\quad\cos\frac{\pi}{2} = 0$ $\quad\tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. *a.)* $\sin\pi$ *b.)* $\cos\frac{3\pi}{2}$ *c.)* $\sin\left(-\frac{\pi}{2}\right)$ *d.)* $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos\frac{\pi}{4}$ *f.)* $\cos(-\pi)$ g) $\cos\frac{\pi}{3}$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ j) $\tan\frac{\pi}{4}$ k) $\tan\pi$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ n) $\sin\frac{11\pi}{6}$ o) $\tan\frac{7\pi}{4}$ p) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$

 **Recall $\sin^2 x = (\sin x)^2$

 **Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x-c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x+c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

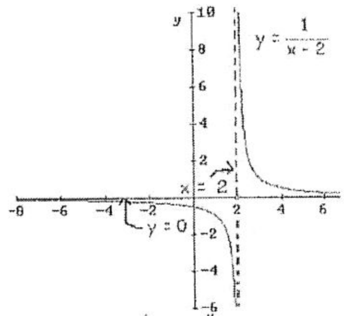
a) $f(x) - 3$ b) $f(x-3)$ c) $2f(x)$ d) $f(x-2)+1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \dfrac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \dfrac{1}{x^2}$

45. $f(x) = \dfrac{x^2}{x^2 - 4}$

46. $f(x) = \dfrac{2+x}{x^2(1-x)}$

47. $f(x) = \dfrac{4-x}{x^2 - 16}$

48. $f(x) = \dfrac{x-1}{x^2 + x - 2}$

49. $f(x) = \dfrac{5x+20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

51. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

52. $f(x) = \frac{4x^2}{3x^2-7}$

53. $f(x) = \frac{(2x-5)^2}{x^2-x}$

54. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

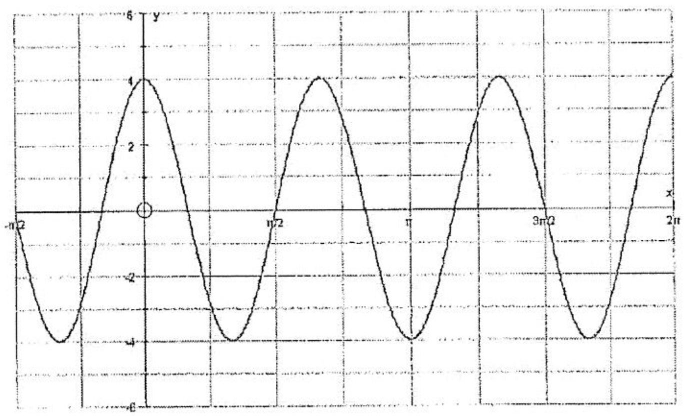
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate $(x, f(x))$ when reflected across the y-axis*)*

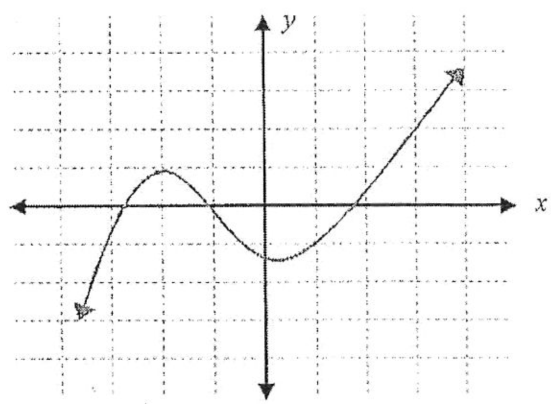
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate $(x, f(x))$ when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ____________________



78. ____________________



79. ____________________
$f(x) = 2x^4 - 5x^2$

80. ____________________
$g(x) = x^5 - 3x^3 + x$

81. ____________________
$h(x) = 2x^2 - 5x + 3$

82. ____________________
$j(x) = 2\cos x$

83. ____________________
$k(x) = \sin x + 4$

84. ____________________
$l(x) = \cos x - 3$

You should not skip this page. We start with an extension of this during Week 2

Finding the Average Rate of Change

Find the average rate of change of $g(t) = t^2 + 3t + 1$ on the interval $[0, a]$. Your answer will be an expression involving a.

Using the average rate of change formula

$$\frac{g(a) - g(0)}{a - 0}$$ Evaluating the function

$$\frac{(a^2 + 3a + 1) - (0^2 + 3(0) + 1)}{a - 0}$$ Simplifying

$$\frac{a^2 + 3a + 1 - 1}{a}$$ Simplifying further, and factoring

$$\frac{a(a + 3)}{a}$$ Cancelling the common factor a

$$a + 3$$

Find the average rate of change of each function on the interval specified. Your answers will be expressions involving a parameter (*b* or *h*).

85. $f(x) = 4x^2 - 7$ on [1, *b*]

86. $h(x) = 3x + 4$ on [2, 2+*h*]

87. $a(t) = \dfrac{1}{t + 4}$ on [9, 9+*h*]

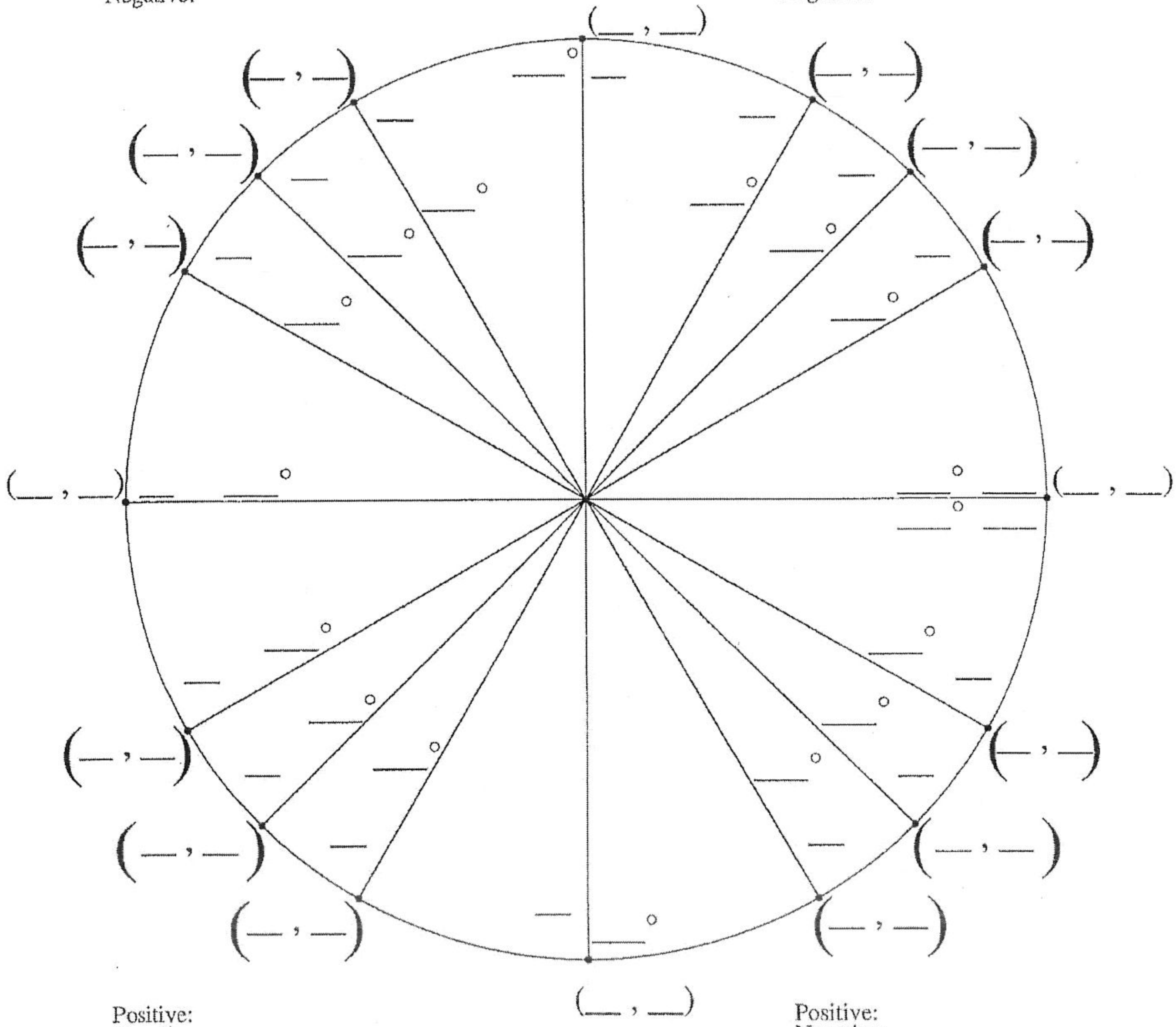
88. $j(x) = 3x^3$ on [1, 1+*h*]

89. $f(x) = 2x^2 + 1$ on [*x*, *x*+*h*]

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative:

EmbeddedMath.com