Calculus P56

(1)

$$\lim_{x \to 1^-} -3x+9 = 6$$

$$\lim_{x \to 1^+} 3x-10 = -7$$

NOT continuous
a
Jump
discontinuity

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(2) (a) Continuous and differentiable

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(b) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(3) $f(7) = \frac{3}{5}$ $f(4) = \frac{3}{2}$

$$\frac{\frac{3}{5} - \frac{3}{2}}{7-4} \quad \frac{\frac{6}{10} - \frac{15}{10}}{3} \quad -\frac{9}{10} \cdot \frac{1}{3} = -\frac{3}{10}$$

(4) $(x+5)^{-1}$ $\quad -1(x+5)^{-2}$

$f'(x) = \dfrac{-1}{(x+5)^2}$ $\quad f'(\cancel{5}\,{-3}) = \dfrac{-1}{2^2} = \dfrac{-1}{4}$

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(5) $y = \frac{1}{4}\,\text{Tan}(6x-4)$

$y' = \frac{1}{4}\left(\text{Sec}^2(6x-4)\right) \cdot 6$

$y' = \frac{3}{2}\left(\text{Sec}(6x-4)\right)^2$

$y'' = 3\,\text{Sec}(6x-4)\,\text{Sec}(6x-4)\,\text{Tan}(6x-4) \cdot 6$

$\boxed{y = 18\,\text{Sec}^2(6x-4) \cdot \text{Tan}(6x-4)}$

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(6) $f(g(x))$ $\quad f'(g(x)) \cdot g'(x)$

$f'(g(4))g'(4)$ $\quad g(4) = 3$ $\quad g'(4) = -6$

$f'(3) = 6$ $\quad 6 \cdot (-6) = -36$

7 $x^2+y^2=7$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y} \qquad \frac{d^2y}{dy^2}=\frac{-\left[y-x\frac{dy}{dx}\right]}{y^2}$$

$$\frac{-y+x\left(-\frac{x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}\cdot\frac{y^2}{y^2}$$

$$\frac{-y^3-x^2y}{y^4}\to\frac{y\left(-y^2-x^2\right)}{y^4}$$

$$\boxed{\frac{-y^2-x^2}{y^3}}$$

(8) $2x^2y - \pi\cos y = 3\pi$ @ $(1, \pi)$

$2\left[x^2\frac{dy}{dx} + 2yx\right] + \pi\sin y\frac{dy}{dx} = 0$

$2\left[\frac{dy}{dx} + 2\pi\right] + \pi\sin\pi\frac{dy}{dx} = 0$

$2\left[\frac{dy}{dx} + 2\pi\right] = 0$ $\quad \frac{dy}{dx} = -2\pi$ — Slope of tangent line

$$\boxed{y - \pi = \frac{1}{2\pi}(x-1)}$$ normal line

(9) $S = T^3 - 15T^2 + 72T$

$S'(T) = 3T^2 - 30T + 72$

$0 = T^2 - 10T + 24$

$0 = (T - \cancel{12})(T \cancel{+} \cancel{2})$ $\quad T = \cancel{12}$ and $T = \cancel{-2}$

6 4

$6(4) - 30$

-6

$S''(T) = 6T - 30$ $\quad 6(-2) - 30$

$S''(T) = 6(\cancel{2}) - 30 \rightarrow \cancel{-12} - 30 \rightarrow$ ~~-42 m/sec²~~

$6(\cancel{-2})$ ~~-42~~

6

(10) $Q(T) = 50(20-x)^2$

$Q'(t) = 100(20-x)(-1)$

$Q'(12) = 100(8)(-1) = -800 \text{ gal/min}$

average $Q(0) = \cancel{0}\ 50(20)^2 = 20000$

$Q(12) = 50(20-12)^2$

$50(8)^2 \to 3200$

$\frac{3200-20000}{12-0}$ $\cancel{266.67 \text{ gal/min}}$ -1400 gal/m