4.1 Solutions to Exercises

1. Linear, because the average rate of change between any pair of points is constant.

3. Exponential, because the difference of consecutive inputs is constant and the ratio of consecutive outputs is constant.

5. Neither, because the average rate of change is not constant nor is the difference of consecutive inputs constant while the ratio of consecutive outputs is constant.

7. $f(x) = 11{,}000(1.085)^x$ You want to use your exponential formula f(x)= ab^x You know the initial value a is 11,000. Since b, your growth factor, is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = 1.085$. This gives you every component of your exponential function to plug in.

9. $f(x) = 23{,}900(1.09)^x$ $f(8) = 47{,}622$. You know the fox population is 23,900, in 2010, so that's your initial value. Since b, your growth factor is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = 1.09$. This gives you every component of your exponential function and produces the function $f(x) = 23{,}900(1.09)^x$. You're trying to evaluate the fox population in 2018, which is 8 years after 2010, the time of your initial value. So if you evaluate your function when $x = 8$, because $2018 - 2010 = 8$, you can estimate the population in 2018.

11. $f(x) = 32{,}500(.95)^x$ $f(12) = \$17{,}561.70$. You know the value of the car when purchased is 32,500, so that's your initial value. Since your growth factor is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = .95$ This gives you every component of your exponential function produces the function $f(x) = 32{,}500(.95)^x$. You're trying to evaluate the value of the car 12 years after it's purchased. So if you evaluate your function when $x = 12$, you can estimate the value of the car after 12 years.

13. We want a function in the form $f(x) = ab^x$. Note that $f(0) = ab^0 = a$; since (0, 6) is a given point, $f(0) = 6$, so we conclude $a = 6$. We can plug the other point (3, 750), into $f(x) = 6b^x$ to solve for b: $750 = 6(b)^3$. Solving gives $b = 5$, so $f(x) = 6(5)^x$.

15. We want a function in the form $f(x) = ab^x$. Note that $f(0) = ab^0 = a$; since (0, 2000) is a given point, $f(0) = 2000$, so we conclude $a = 2000$. We can plug the other point (2, 20) into $f(x) = 2000b^x$, giving $20 = 2000(b)^2$. Solving for b, we get $b = 0.1$, so $f(x) = 2000(.1)^x$.

17. $f(x) = 3(2)^x$ For this problem, you are not given an initial value, so using the coordinate points your given, $\left(-1, \frac{3}{2}\right), (3, 24)$ you can solve for b and then a. You know for the first coordinate point, $\left(\frac{3}{2}\right) = a(b)^{-1}$. You can now solve for a in terms of b: $\left(\frac{3}{2}\right) = \frac{a}{b} \rightarrow \left(\frac{3b}{2}\right) = a$. Once you know this, you can substitute $\left(\frac{3b}{2}\right) = a$, into your general equation, with your other coordinate point, to solve for b: $24 = \left(\frac{3b}{2}\right)(b)^3 \rightarrow 48 = 3b^4 \rightarrow 16 = b^4 \rightarrow b = 2$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points your given: $24 = a(2)^3 \rightarrow 24 = 8a \rightarrow a = 3$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = 3(2)^x$.

19. $f(x) = 2.93(.699)^x$ For this problem, you are not given an initial value, so using the coordinate points you're given, $(-2,6), (3,1)$ you can solve for b and then a. You know for the first coordinate point, $1 = a(b)^3$. You can now solve for a in terms of b: $\frac{1}{b^3} = a$. Once you know this, you can substitute $\frac{1}{b^3} = a$, into your general equation, with your other coordinate point, to solve for b: $6 = \frac{1}{b^3}(b)^{-2} \rightarrow 6b^5 = 1 \rightarrow b^5 = \frac{1}{6} \rightarrow b = .699$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points you're given: $6 = a(.699)^{-2} \rightarrow$ 6=2.047a $\rightarrow a = 2.93$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = 2.93(.699)^x$

21. $f(x) = \frac{1}{8}(2)^x$ For this problem, you are not given an initial value, so using the coordinate points you're given, $(3,1), (5,4)$ you can solve for b and then a. You know for the first coordinate point, $1 = a(b)^3$. You can now solve for a in terms of b: $1/b^3 = a$. Once you know this, you can substitute $\frac{1}{b^3} = a$, into your general equation, with your other coordinate point, to solve for b: $4 = \frac{1}{b^3}(b)^5 \ \rightarrow \ 4 = b^2 \ \rightarrow \ b = 2$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points your given: $1 = a(2)^3 \ \rightarrow 1 = 8a \rightarrow \ a = 1/8$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = \frac{1}{8}(2)^x$

23. 33.58 milligrams. To solve this problem, you want to use the exponential growth/decay formula , $f(x) = a(b)^x$, to solve for b, your growth factor. Your starting amount is a, so a=100 mg. You are given a coordinate, (35,50), which you can plug into the formula to solve for b, your effective growth rate giving you your exponential formula $f(x) = 100(0.98031)^x$ Then you can plug in your $x = 54,$ to solve for your substance.

25. $1,555,368.09 Annual growth rate: 1.39% To solve this problem, you want to use the exponential growth/decay formula f(x)=ab^x First create an equation using the initial conditions, the price of the house in 1985, to solve for a. You can then use the coordinate point you're given to solve for b. Once you've found a, and b, you can use your equation f(x)=$110{,}000(1.0139)^x$ to predict the value for the given year.

27. $4,813.55 To solve this problem, you want to use the exponential growth/decay formula f(x)=ab^x First create an equation using the initial conditions, the value of the car in 2003, to solve for a. You can then use the coordinate point you're given to solve for b. Once you've found a, and b, you can use your equation f(x)=$38{,}000(.81333)^x$ to predict the value for the given year.

29. Annually: $7353.84 Quarterly: $47469.63 Monthly: $7496.71 Continuously: $7,501.44. Using the compound interest formula A(t)=$a(1 + \frac{r}{K})^{Kt}$ you can plug in your starting amount,

\$4000 to solve for each of the three conditions, annually—$k = 1$, quarterly—$k = 4$, and monthly—$k = 12$. You then need to plug your starting amount, \$4000 into the continuous growth equation f(x)=ae^{rx} to solve for continuous compounding.

31. APY= .03034 ≈ 3.03% You want to use the APY formula $f(x) = (1 + \frac{r}{K})^K$-1 you are given a rate of 3% to find your r and since you are compounding quarterly K=4

33. $t = 7.4$ years To find out when the population of bacteria will exceed 7569 you can plug that number into the given equation as P(t) and solve for t. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 1600, then take the ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

35. (a) $w(t) = 1.1130(1.0464)^t$ For this problem, you are not given an initial value, since 1960 corresponds to 0, 1968 would correspond to 8 and so on, giving you the points $(8{,}1.60)$ $(16{,}2.30)$ you can use these points to solve for b and then a. You know for the first coordinate point, $1.60 = ab^8$. You can now solve for a in terms of b: $\frac{1.60}{b^8} = a$. Once you know this, you can substitute $\frac{1.60}{b^8} = a$, into your general equation, with your other coordinate point, to solve for b: $2.30 = \frac{1.60}{b^8}(b)^{16} \rightarrow 1.60b^8 = 2.30 \rightarrow b^8 = \frac{2.30}{1.60} \rightarrow b = 1.0464$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points you're given: $2.30 = a(1.0464)^{16} \rightarrow$ $2.30 = 2.0664a \rightarrow a = 1.1130$. So now that you've solved for a and b, you can come up with your general equation: $w(t) = 1.1130(1.0464)^t$

(b) \$1.11 using the equation you found in part a you can find w(0)

(c) The actual minimum wage is less than the model predicted, using the equation you found in part a you can find w(36) which would correspond to the year 1996

37. (a) 512 dimes the first square would have 1 dime which is 2^0 the second would have 2 dimes which is 2^1 and so on, so the tenth square would have 2^9 or 512 dimes

(b) 2^{n-1} if n is the number of the square you are on the first square would have 1 dime which is 2^{1-1} the second would have 2 dimes which is 2^{2-1} the fifteenth square would have 16384 dimes which is 2^{15-1}

(c) 2^{63}, 2^{64-1}

(d) 9,223,372,036,854,775,808 mm

(e) There are 1 million millimeters in a kilometer, so the stack of dimes is about 9,223,372,036,855 km high, or about 9,223,372 million km. This is approximately 61,489 times greater than the distance of the earth to the sun.