Sep D.E.Q

(1) $\frac{dy}{dx} = \frac{x}{y}$ $\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$\boxed{y^2 = x^2 + C}$

(2) $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$ $3\int y^2\,dy = \int x^2 + 2\,dx$

$y^3 = x^3 + 2x + C$

$\boxed{y = \sqrt[3]{x^3+2x+c}}$

(3) $\frac{dy}{dx} = \frac{y}{x}$ $\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + C$

$y = e^{\ln x + C} \to e^{C}e^{\ln x}$

$\boxed{y = ax}$

(4) $(2+x)\dfrac{dy}{dx} = 3y$

$$\int \frac{dy}{y} = \int \frac{3}{2+x}\,dx$$

$$\ln y = 3\ln(2+x) + C$$

$$\ln y = \ln(2+x)^3 + C$$

$$\boxed{y = a(2+x)^3} \qquad \boxed{x > -2}$$

(5) $y\dfrac{dy}{dx} = \sin x$

$$\int y\,dy = \int \sin x\,dx$$

$$\frac{1}{2}y^2 = -\cos x + C$$

$$\boxed{y^2 = -2\cos x + C}$$

(4) $\frac{dy}{dx} = (1+4y^2)(1+3x^2)$

$$\int \frac{dy}{1+4y^2} = \int 1+3x^2 \, dy$$

I think this is $\frac{1}{2}(\arctan(2y))$

$$\frac{1}{2}\arctan(2y) = x + x^3 + C$$

$$\arctan 2y = 2x + 2x^3 + C$$

$$2y = \tan(2x + 2x^3 + C)$$

$$\boxed{y = \frac{1}{2}\tan(2x + 2x^3 + C)}$$

Wolfram Alpha agrees.

(7) $\dfrac{dy}{dx} = \dfrac{-\sqrt{x}}{\sqrt{y}}$ $y(1) = 4$

$\sqrt{y}\, dy = -\sqrt{x}\, dy$

$\int y^{\frac{1}{2}} dy = -\int x^{\frac{1}{2}} dx$

$\frac{2}{3} y^{3/2} = -\frac{2}{3} x^{3/2} + C$

$\frac{2}{3}(4)^{3/2} = -\frac{2}{3} + C$

$\frac{2}{3}(8) = -\frac{2}{3} + C$

$\frac{16}{3} + \frac{2}{3} = C$

$\frac{18}{3} = 6 \rightarrow C$

$\frac{2}{3} y^{\frac{3}{2}} = -\frac{2}{3} x^{3/2} + 6$

$y^{3/2} = -x^{3/2} + 9$

$\boxed{y = \left(\sqrt[3]{9 - x^{3/2}}\right)^2}$

(8) $\frac{dy}{dx} = \frac{e^x}{y}$ $\qquad y(0) = 4$

$\int y\,dy = \int e^x\,dx$

$\frac{1}{2}y^2 = e^x + C$

$\frac{1}{2}(4)^2 = e^0 + C$

$8 = 1 + C$

$7 = C$

$\frac{1}{2}y^2 = e^x + 7$

$y^2 = 2e^x + 14$

$\boxed{y = \sqrt{2e^x + 14}}$

we know $+\sqrt{\ }$ b/c

(4)

$$\frac{dy}{dx} = \frac{\ln x}{xy}$$

needs a u sub. To integrate Sec 6.2

$$y\,dy = \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int y\,dy = \int u\,du$$

$$\frac{1}{2}y^2 = \frac{1}{2}u^2 + C$$

$$\frac{1}{2}y^2 = \frac{1}{2}(\ln x)^2 + C \qquad (1,0)$$

$$0 = \frac{1}{2}(\ln 1)^2 + C$$

$$0 = \frac{1}{2}(0) + C$$

$$0 = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}(\ln(x))^2$$

$$y^2 = (\ln x)^2$$

$$y = \ln x$$

(10) $$\frac{dT}{dT} = -k(T-70) \quad T(0)=140$$

$$\frac{dT}{T-70} = -k\,dT$$

$$\ln(T-70) = -kT + C$$

$$T-70 = ae^{-kT}$$

$$T = ae^{-kT} + 70 \quad (0, 140)$$

$$140 = ae^{0} + 70$$

$$70 = a$$

$$\boxed{T = 70e^{-kT} + 70}$$