Chapter 6 Notes (Sec 6.1)

Pg 2 d

(11) $y = 3\sin(8(x+4)) + 5$

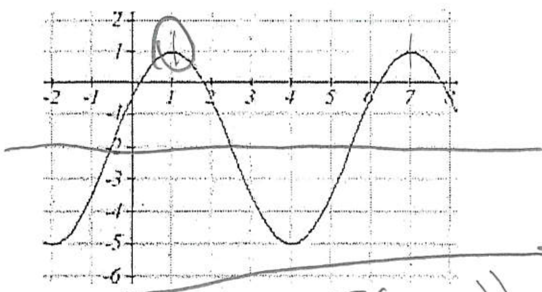
	Amp	period	phase shift	midline
11	3	$\frac{2\pi}{8}$ $\boxed{\frac{\pi}{4}}$	Left 4	$y=5$
12	4	$\frac{2\pi}{\frac{\pi}{2}}$ $\frac{2\pi}{1} \cdot \frac{2}{\pi}$ $\boxed{4}$	Right 3	$y=7$
13	2	$3(x-7)$ $\boxed{\frac{2\pi}{3}}$	Right 7	$y=4$
14	5	$5(x+4)$ $\boxed{\frac{2\pi}{5}}$	Left 4	$y=-2$
15	1	12 see sheet	Left 6	$y=-3$
16	8	$\frac{12}{7}$	Left 3	$y=6$

e

$\frac{2\pi}{6} = \frac{\pi}{3}$

Second Video

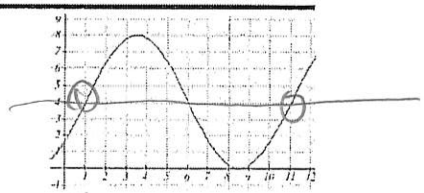
Writing a function from the graph of a transformed sinusoidal function



$\boxed{3\cos\left(\frac{\pi}{3}(x-1)\right)-2}$

2)

Write a formula for the function graphed here.



$\frac{2\pi}{10}$

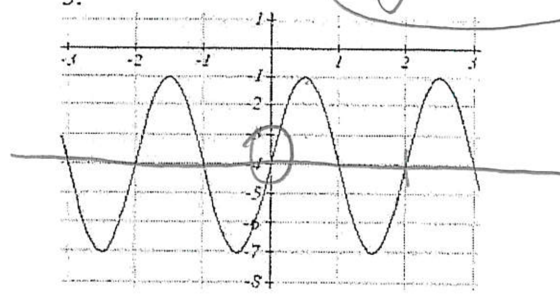
$y = 4\sin\left(\frac{\pi}{5}(x-1)\right)+4$

f

$\frac{2\pi}{2} = \pi$

$y = 3\sin(\pi x) - 4$

5.



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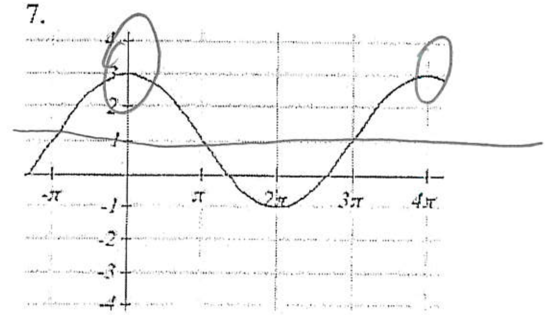


$\frac{2\pi}{4\pi} = \frac{1}{2}$

$\boxed{y = 2\cos\left(\frac{1}{2}x\right)+1}$

C

Try it Now

3. If a sinusoidal function starts on the midline at point (0,3), has an amplitude of 2, and a period of 4, write a formula for the function.

Example 10

Sketch a graph of $f(t) = 3\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$.

A

For each of the following equations, find the amplitude, period, horizontal shift, and midline.

11. $y = 3\sin(8(x+4)) + 5$

12. $y = 4\sin\left(\frac{\pi}{2}(x-3)\right) + 7$

13. $y = 2\sin(3x - 21) + 4$

14. $y = 5\sin(5x + 20) - 2$

15. $y = \sin\left(\frac{\pi}{6}x + \pi\right) - 3$

16. $y = 8\sin\left(\frac{7\pi}{6}x + \frac{7\pi}{2}\right) + 6$

$\frac{\frac{\pi}{6}x + \pi}{\frac{\pi}{6}}$ $\quad \frac{\pi}{\frac{\pi}{6}}$ $\quad \frac{\pi}{6}(x+6)$

$\frac{\cancel{\pi}}{1} \cdot \frac{6}{\cancel{\pi}}$

Period

$\frac{2\pi}{1} \div \frac{\pi}{6}$

$\frac{2\cancel{\pi}}{1} \cdot \frac{6}{\cancel{\pi}} = 12$

$\frac{7\pi}{6} \quad \frac{7\pi}{2}$

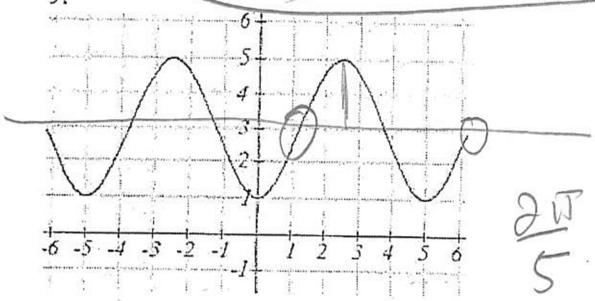
$y = 8\sin\left(\frac{7\pi}{6}(x+3)\right) + 6$

$\frac{7\pi}{2} \cdot \frac{6}{7\pi} = 3$ Period $\frac{2\pi}{1} \cdot \frac{6}{7\pi}$ $\frac{12}{7}$

G

9.

$y = 2\sin\left(\frac{2\pi}{5}(x-1)\right)+3$



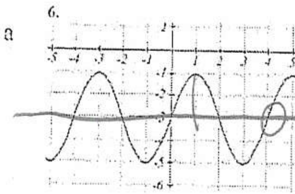
$\frac{2\pi}{5}$

Exercises

For the graphs below, determine the amplitude, midline, and period, then find a formula for the function.

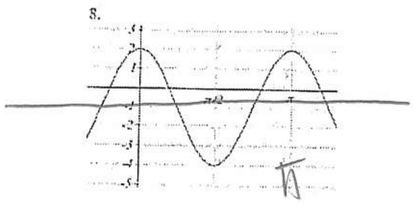
$\frac{2\pi}{4} = \frac{\pi}{2}$

6.



$y=2\sin\left(\frac{\pi}{2}x\right) - 3$

8.



π

$\boxed{y = 3\cos(2x) - 1}$

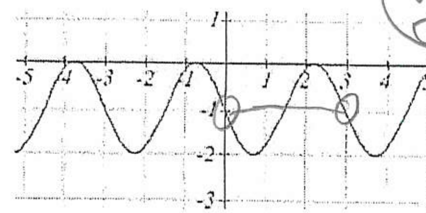
(4)

H

10.

$\frac{2\pi}{3}$

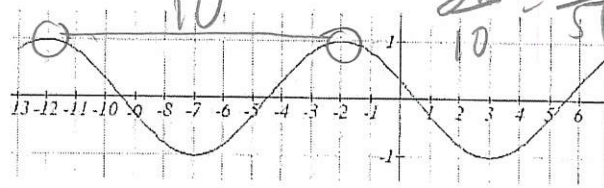
$y = -\sin\left(\frac{2\pi}{3}x\right) - 1$



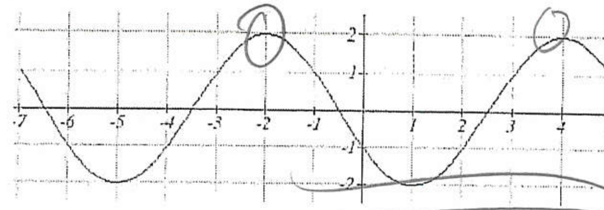
10

$\frac{2\pi}{10} = \frac{\pi}{5}$

$y = \cos\left(\frac{\pi}{5}(x+2)\right)$



$\frac{2\pi}{6}$



$y = 2\cos\left(\frac{\pi}{3}(x+2)\right)$

Exercises

Find the period and horizontal shift of each of the following functions.

5. $f(x) = 2\tan(4x - 32)$

$2\tan(4(x-8))$ Per

7. $h(x) = 2\sec\left(\frac{\pi}{4}(x+1)\right)$

$\frac{2\pi}{\frac{\pi}{4}} \to \frac{2\pi}{1} \cdot \frac{4}{\pi}$

9. $m(x) = 6\csc\left(\frac{\pi}{3}x + \pi\right)$

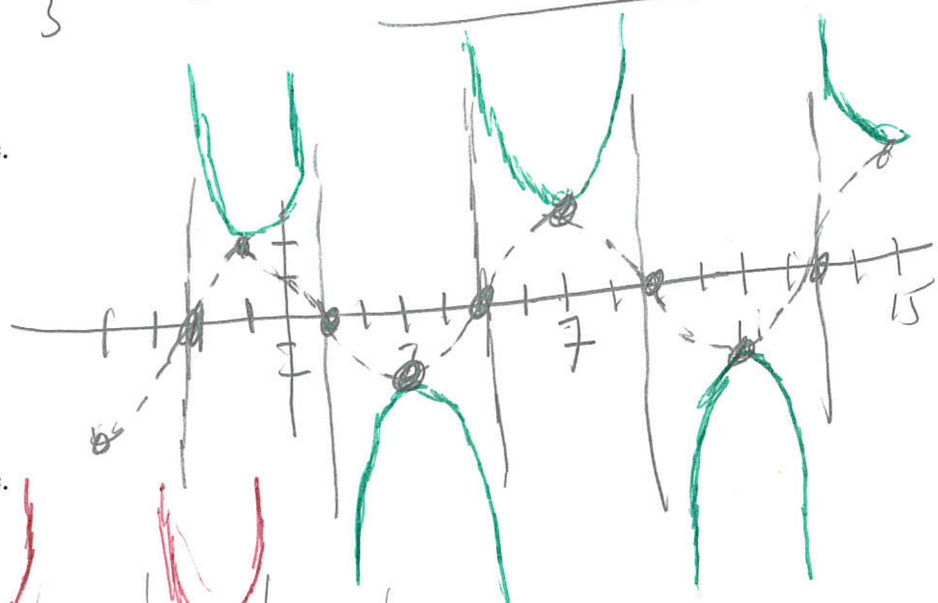
$\frac{2\pi}{\frac{\pi}{3}}$ $\frac{2\pi}{1} \cdot \frac{3}{\pi}$

$6\csc\left(\frac{\pi}{3}(x+3)\right)$

Per	Phase Shift
$\frac{\pi}{4}$	Rt 8
8	Left 1
6	Left 3

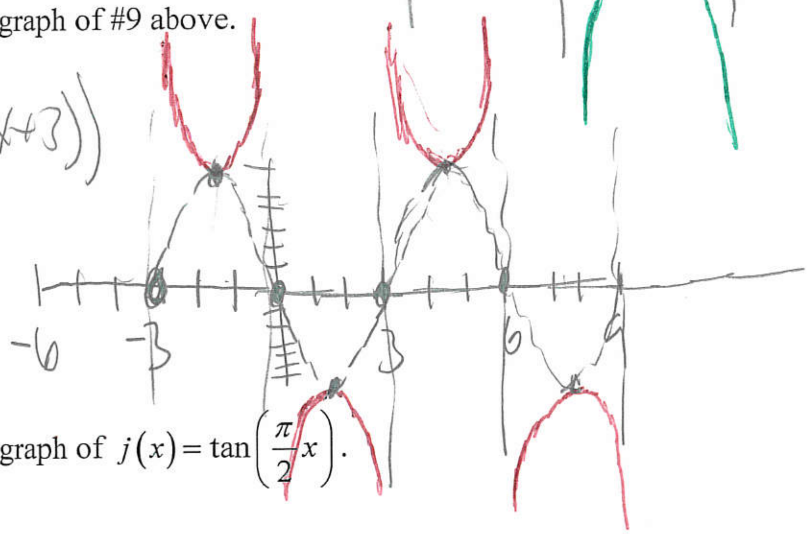
11. Sketch a graph of #7 above.

$2\cos\left(\frac{\pi}{4}(x+1)\right)$



13. Sketch a graph of #9 above.

$6\sin\left(\frac{\pi}{3}(x+3)\right)$



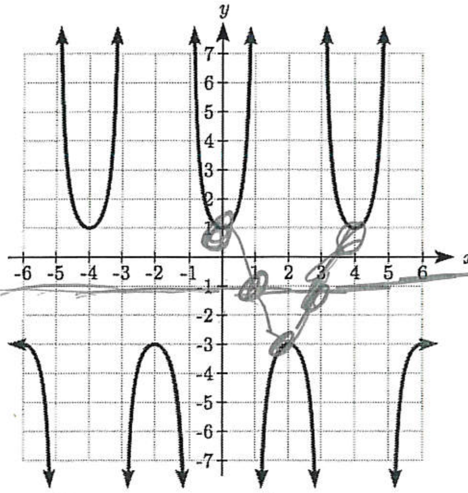
15. Sketch a graph of $j(x) = \tan\left(\frac{\pi}{2}x\right)$.

9

Exercises

Find a formula for each function graphed below.

CSC

17.


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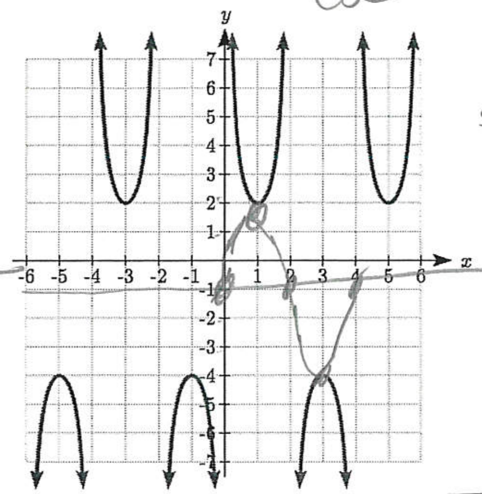


$\frac{2\pi}{4}$ $\frac{\pi}{2}$

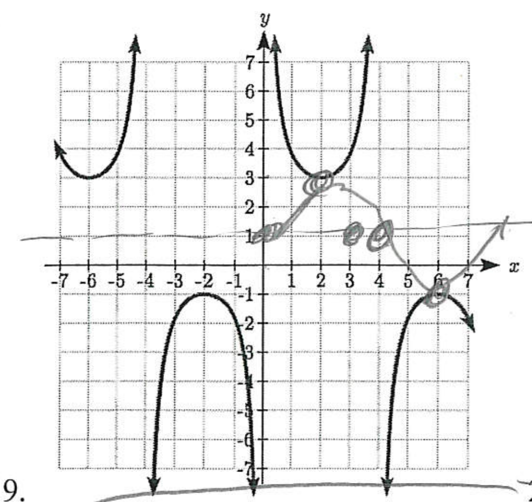
A

Per=4 $\frac{2\pi}{4}$ $\frac{\pi}{2}$

Amp=2 mid line $y=-2$

$\boxed{y = 2\sec\left(\frac{\pi}{2}x\right) - 2}$

$\boxed{y = 3\csc\left(\frac{\pi}{2}x\right) - 1}$

19.


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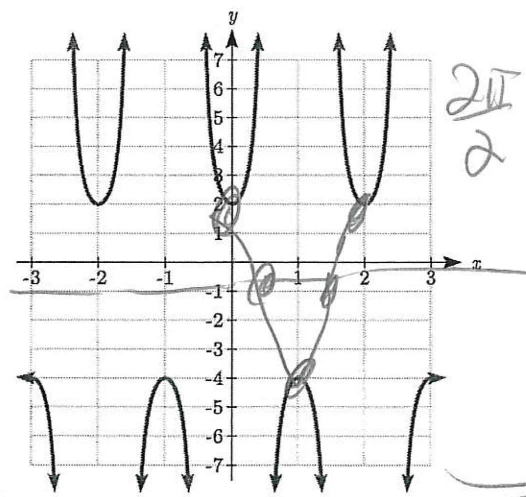


$\frac{2\pi}{2}$

$y = 2\csc\left(\frac{\pi}{4}x\right) + 1$

$3\sec(\pi x) - 1$

10

$\sec x = \text{even}$ $\tan x = \text{odd}$

Exercises $\csc x = \text{odd}$

21. If $\tan x = -1.5$, find $\tan(-x)$. 1.5

22. If $\tan x = 3$, find $\tan(-x)$. -3

23. If $\sec x = 2$, find $\sec(-x)$. 2

24. If $\sec x = -4$, find $\sec(-x)$. -4

25. If $\csc x = -5$, find $\csc(-x)$. 5

26. If $\csc x = 2$, find $\csc(-x)$. -2

Simplify each of the following expressions completely.

27. $\cot(-x)\cos(-x) + \sin(-x)$

$$-\frac{\cos x}{\sin x} \cdot \cos x - \sin x \quad \frac{-\cos^2 x - \sin^2 x}{\sin x} \quad \frac{-1}{\sin x} \quad \boxed{-\csc x}$$

28. $\cos(-x) + \tan(-x)\sin(-x)$

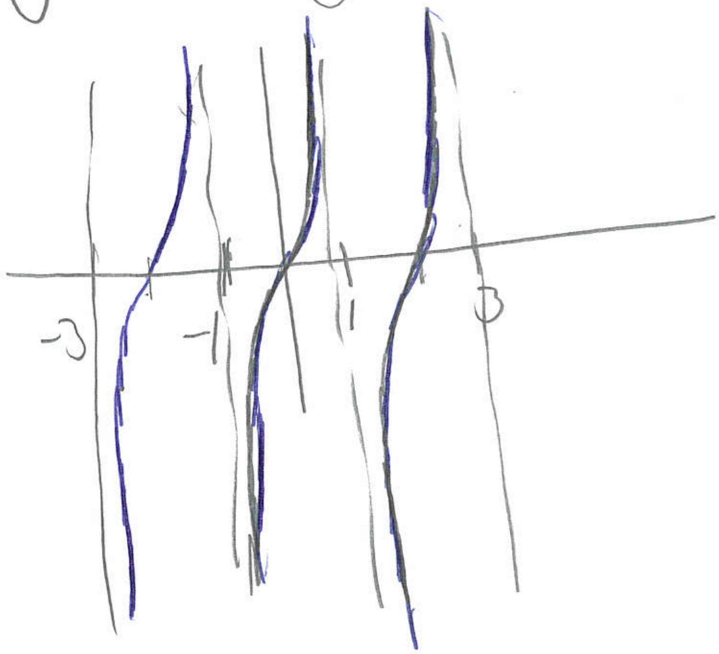
$$\frac{\cos x}{1} - \frac{\sin x}{\cos x} \cdot (-\sin x)$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x} \quad \frac{1}{\cos x} \rightarrow \boxed{\sec x}$$

11

Pg #15

$y = \tan\left(\frac{\pi}{2}x\right)$ Per $\frac{\pi}{\frac{\pi}{2}} \to \frac{\cancel{\pi}}{1} \cdot \frac{2}{\cancel{\pi}} = 2$



Inverse Trig Functions - Homework

.1. Evaluate each of the following:

a. $\arccos\frac{1}{2}$ $\quad \frac{\pi}{6}$

$\cos x = \frac{1}{2}$

b. $\cot^{-1}\sqrt{3}$ $\quad \frac{\pi}{3}$

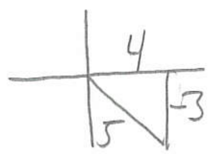
$\cot x = \frac{\sqrt{3}}{1}$

c. $\sin^{-1}\frac{-\sqrt{3}}{2}$ $\quad -\frac{\pi}{3}$

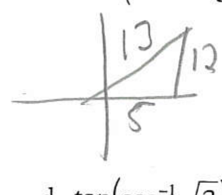
d. $\sec^{-1}-\frac{2\sqrt{3}}{3}$ $\quad -\frac{2}{\sqrt{3}}$ $\quad \frac{2\pi}{3}$

2. Evaluate the following. Make a picture to describe the situation.

a. $\cos\left(\arcsin\frac{-3}{5}\right)$ $\quad \frac{4}{5}$

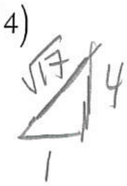


b. $\sin\left(\arctan\frac{12}{5}\right)$ $\quad \frac{12}{13}$



c. $\csc\left(\cot^{-1}4\right)$

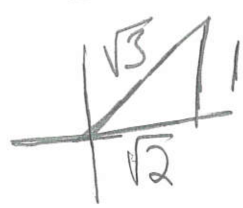
$\cot x = \frac{4}{1}$



$\frac{\sqrt{17}}{4}$

d. $\tan\left(\csc^{-1}\sqrt{3}\right)$

$\csc x = \frac{\sqrt{3}}{1}$



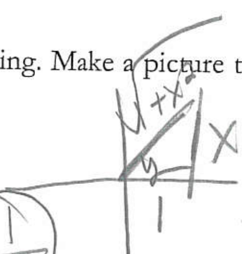
$\frac{\sqrt{2}}{2}$

3. Evaluate the following. Make a picture to describe the situation.

a. $\cos\left(\tan^{-1}x\right)$

$y = \tan^{-1}x$

$\tan y = \frac{x}{1}$ $\quad \frac{1}{\sqrt{1+x^2}}$

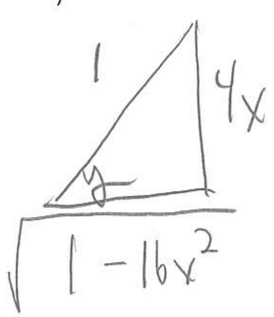


b. $\sec\left(\sin^{-1}x\right)$ $\quad \sin y = \frac{x}{1}$

$\frac{1}{\sqrt{1-x^2}}$

c. $\tan\left(\sin^{-1}4x\right)$

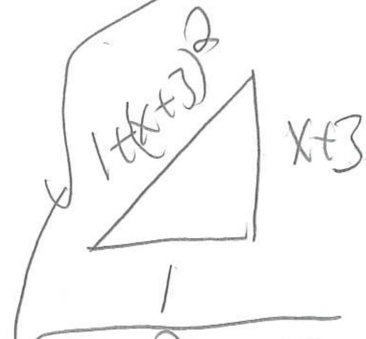
$\sin(y) = \frac{4x}{1}$



$\frac{4x}{\sqrt{1-16x^2}}$

d. $\cos\left(\tan^{-1}(x+3)\right)$

$\tan y = \frac{x+3}{1}$



$\sqrt{1+x^2+6x+9}$

$\cos y = \sqrt{x^2+6x+10}$

15 $\quad \frac{1}{\sqrt{x^2+6x+10}}$

Use your calculator to evaluate each expression, giving the answer in radians. If there is an error, explain why.

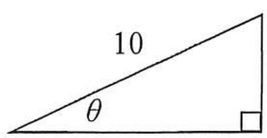
13. $\cos^{-1}(-0.4)$ 14. $\cos^{-1}(0.8)$ 15. $\sin^{-1}(-0.8)$ 16. $\tan^{-1}(6)$

1.98 0.644 −0.927 1.407

17. $cos^{-1}(1.56)$

NOT Possible $\cos x \leq 1$

Find the angle θ in degrees.



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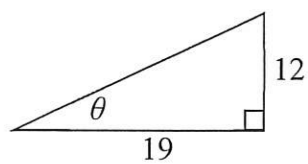


19.

$\arcsin\left(\frac{7}{10}\right)$ $\arctan\left(\frac{12}{19}\right)$

16

Section 6.3 Exercises

Evaluate the following expressions, giving the answer in radians.

1. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\frac{\pi}{4}$

3. $\sin^{-1}\left(-\frac{1}{2}\right)$ $-\frac{\pi}{6}$

5. $\cos^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{3}$

7. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $-\frac{\pi}{4}$

9. $\tan^{-1}(1)$ $\frac{\pi}{4}$

11. $\tan^{-1}\left(-\sqrt{3}\right)$ 1 2 $\sqrt{3}$ $-\frac{\pi}{3}$

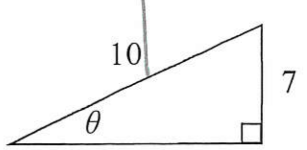
Use your calculator to evaluate each expression, giving the answer in radians.

13. $\cos^{-1}(-0.4)$

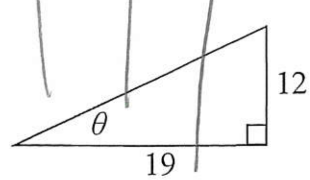
15. $\sin^{-1}(-0.8)$

16. $\tan^{-1}(6)$

Find the angle θ in degrees.



17.



18.

17

17

Evaluate the following expressions.

19. $\sin^{-1}\left(\cos\left(\frac{\pi}{4}\right)\right)$

$\sin y = \cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$

21. $\sin^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

$\sin y = \cos\frac{4\pi}{3}$ $\arcsin\left(-\frac{1}{2}\right)$

$\sin y = -\frac{1}{2}$ $-\frac{\pi}{6}$

23. $\cos\left(\sin^{-1}\left(\frac{3}{7}\right)\right)$

$\frac{2\sqrt{10}}{7}$

$\sin y = \frac{3}{7}$

$\sqrt{40}$ $2\sqrt{10}$

25. ~~cos~~ $\cos(\tan^{-1}(4))$

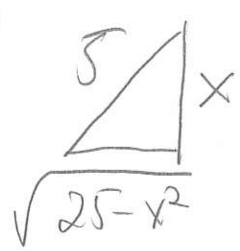
$\tan y = \frac{4}{1}$

$\frac{1}{\sqrt{17}}$ or $\frac{\sqrt{17}}{17}$

Find a simplified expression for each of the following.

27. $\sin\left(\cos^{-1}\left(\frac{x}{5}\right)\right)$, for $-5 \le x \le 5$

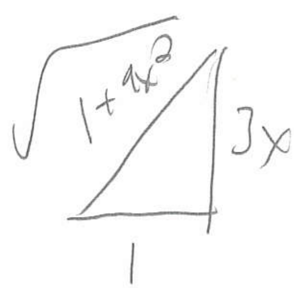
$\cos y = \frac{x}{5}$



$\sin y = \frac{x}{\sqrt{25-x^2}}$

29. $\sin(\tan^{-1}(3x))$

$\tan y = \frac{3x}{1}$



$\sin y = \frac{3x}{\sqrt{1+9x^2}}$

18

Section 6.4 Exercises

Give all answers in radians unless otherwise indicated.

Find all solutions on the interval $0 \le \theta < 2\pi$.

1. $2\sin(\theta) = -\sqrt{2}$

$\sin\theta = -\frac{\sqrt{2}}{2}$ $\quad \frac{5\pi}{4}$ and $\frac{7\pi}{4}$

3. $2\cos(\theta) = 1$

$\cos\theta = \frac{1}{2}$ $\quad \frac{\pi}{3}$ and $\frac{5\pi}{3}$

5. $\sin(\theta) = 1$

$\frac{\pi}{2}$

7. $\cos(\theta) = 0$ $\quad \frac{\pi}{2}$ and $\frac{3\pi}{2}$

Find all solutions.

9. $2\cos(\theta) = \sqrt{2}$ $\quad \frac{\pi}{4} + 2\pi k$ and $\frac{7\pi}{4} + 2\pi k$

$\cos\theta = \frac{\sqrt{2}}{2}$

11. $2\sin(\theta) = -1$

$\sin\theta = -\frac{1}{2}$ $\quad \frac{7\pi}{6} + 2\pi k$ $\quad \frac{11\pi}{6} + 2\pi k$

Find all solutions.

13. $2\sin(3\theta) = 1$ $\quad 3\theta = \frac{\pi}{6} + 2\pi k$ $\quad 3\theta = \frac{5\pi}{6} + 2\pi k$

$\sin 3\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{18} + \frac{2\pi}{3}k$ $\quad \theta = \frac{5\pi}{18} + \frac{2\pi}{3}k$

15. $2\sin(3\theta) = -\sqrt{2}$

$\sin(3\theta) = -\frac{\sqrt{2}}{2}$ $\quad 3\theta = \frac{5\pi}{4} + 2\pi k$ $\quad 3\theta = \frac{7\pi}{4} + 2\pi k$

$\theta = \frac{5\pi}{12} + \frac{2\pi}{3}k$ $\quad$ 27 $\quad \theta = \frac{7\pi}{12} + \frac{2\pi}{3}k$

6.4 key

17. $2\cos(2\theta)=1$

$\cos(2\theta)=\frac{1}{2}$

$2\theta=\frac{\pi}{3}+2\pi k$ $\quad$ $2\theta=\frac{5\pi}{3}+2\pi k$

$\boxed{\theta=\frac{\pi}{6}+\pi k}$ $\quad$ $\boxed{\theta=\frac{5\pi}{6}+\pi k}$

19. $2\cos(3\theta)=-\sqrt{2}$

$\cos(3\theta)=-\frac{\sqrt{2}}{2}$

$3\theta=\frac{3\pi}{4}+2\pi k$ $\quad$ $3\theta=\frac{5\pi}{4}+2\pi k$

$\boxed{\theta=\frac{\pi}{4}+\frac{2\pi}{3}k}$ $\quad$ $\boxed{\theta=\frac{5\pi}{12}+\frac{2\pi}{3}k}$

21. $\cos\left(\frac{\pi}{4}\theta\right)=-1$

$\frac{\pi}{4}\theta=\pi+2\pi k$ $\quad$ $\theta=\frac{4}{\pi}(\pi+2\pi k)\rightarrow\boxed{\theta=4+8k}$

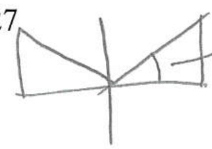
23. $2\sin(\pi\theta)=1.$ $\quad$ $\sin(\pi\theta)=\frac{1}{2}$

$\pi\theta=\frac{\pi}{6}+2\pi k$ $\quad$ $\pi\theta=\frac{5\pi}{6}+2\pi k$

$\boxed{\theta=\frac{1}{6}+2k}$ $\quad$ $\boxed{\theta=\frac{5}{6}+2k}$

Find all solutions on the interval $0\le x<2\pi$.

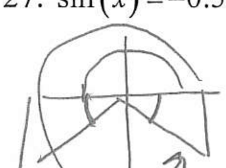
25. $\sin(x)=0.27$



arcsin(.27) $\quad$ $\pi-\arcsin(.27)$

.273 $\quad$ $\pi-.273$

27. $\sin(x)=-0.58$



arcsin(−.58) = −.619

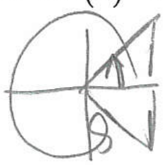
$\pi+.619$ $\quad$ $2\pi-.619$

29. $\cos(x)=-0.55$

arccos(−.55) = 2.153 $\quad$ $\pi-2.153=.988$

$\pi+.988$

31. $\cos(x)=0.71$

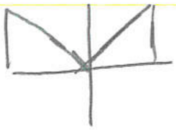


arccos(.71) = .781 $\quad$ $2\pi-.781$

28

Find the first two positive solutions.

33. $7\sin(6x) = 2$



$\sin(6x) = \frac{2}{7}$

$\arcsin\left(\frac{2}{7}\right) = .2898 + 2\pi k$

$\pi - .2898 = 2.852 + 2\pi k$

$6x = .2898 + 2\pi k$

$x = .0483 + \frac{\pi}{3}k$

$x = .475 + \frac{\pi}{3}k$

.0483 and .475

35. $5\cos(3x) = -3$

$\cos(3x) = -\frac{3}{5}$

$3x = 2.21 + 2\pi k$

$x = .738 + \frac{2\pi}{3}k$

$\pi - 2.21$

$3x = \pi + .927 + 2\pi k$

$3x = 4.069 + 2\pi k$

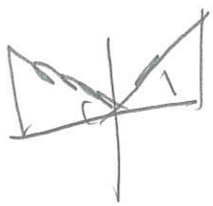
$x = 1.356 + \frac{2\pi}{3}k$

37. $3\sin\left(\frac{\pi}{4}x\right) = 2$

39. $5\cos\left(\frac{\pi}{3}x\right) = 1$

37

$\arcsin\left(\frac{2}{3}\right)$



$\frac{\pi}{4}x = .7297 + 2\pi k$

$\boxed{x = .9291 + 8k}$

and

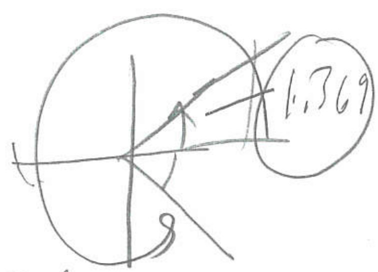
$\frac{\pi}{4}x = 2.412 + 2\pi k$

$x = 3.071 + 8k$

$\boxed{.9291 \text{ and } 3.071}$

39

$\arccos \frac{1}{5}$



$2\pi - 1.369$

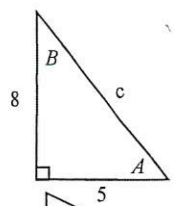
4.914

2a

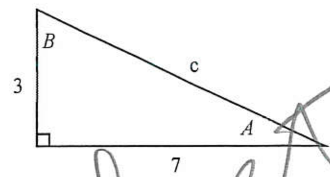
Section 6.5 Exercises

In each of the following triangles, solve for the unknown side and angles.

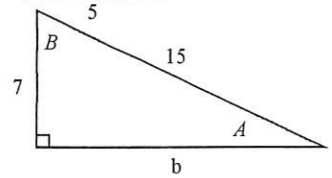
1.



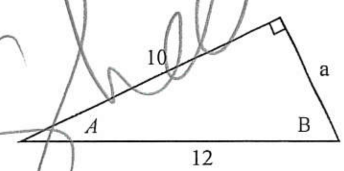
2.



3.



4.



Find a possible formula for the trigonometric function whose values are in the following tables.

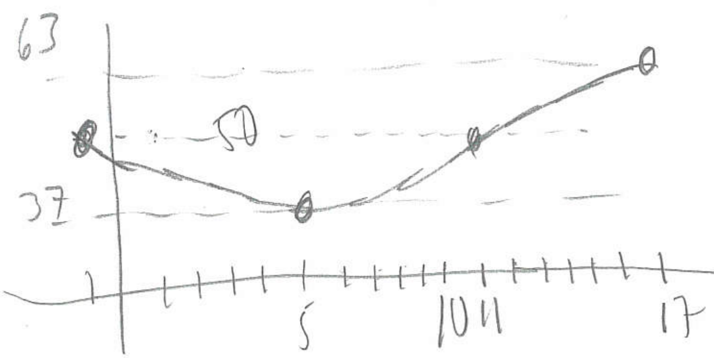
5.

x	0	1	2	3	4	5	6
y	-2	4	10	4	-2	4	10

6.

x	0	1	2	3	4	5	6
y	1	-3	-7	-3	1	-3	-7

7. Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature for the day is 63 degrees and the low temperature of 37 degrees occurs at 5 AM. Assuming *t* is the number of hours since midnight, find an equation for the temperature, *D*, in terms of *t*.



$\frac{63-37}{2} = \text{Amp}$

$\boxed{-13\sin\left(\frac{\pi}{12}(T+1)\right)+50}$

Per = 24 hrs

$b = \frac{2\pi}{24}$ or $\frac{\pi}{12}$

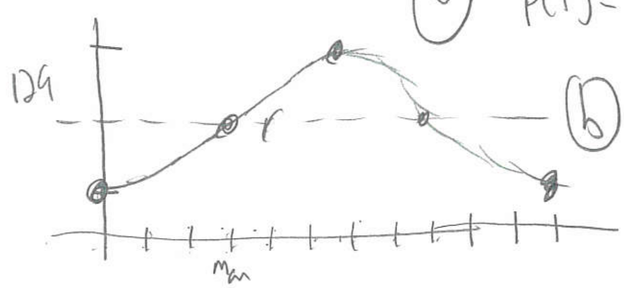
32

9. A population of rabbits oscillates 25 above and below an average of 129 during the year, hitting the lowest value in January ($t = 0$).

a. Find an equation for the population, P, in terms of the months since January, t.
b. What if the lowest value of the rabbit population occurred in April instead?

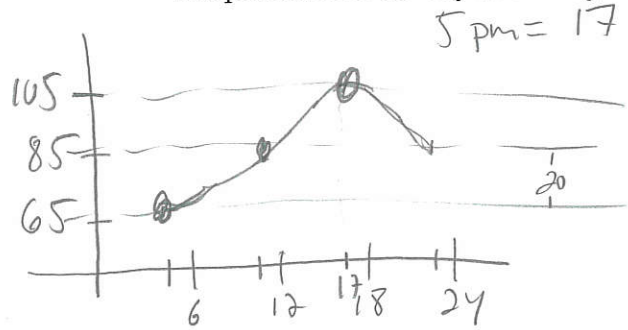
(a) $P(T) = -25\cos\left(\frac{\pi}{6}T\right) + 129$

(b) $P(t) = -25\cos\left(\frac{\pi}{6}(T-4)\right) + 129$



11. Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature of 105 degrees occurs at 5 PM and the average temperature for the day is 85 degrees. Find the temperature, to the nearest degree, at 9 AM.

5 pm = 17



$-20\cos\left(\frac{\pi}{12}(T-5)\right) + 85$

$-20\cos\left(\frac{\pi}{12}(9-5)\right) + 85$

$-20\cos\left(\frac{\pi}{12}\cdot 4\right) + 85$

$-20\cos\frac{\pi}{3} + 85 \rightarrow -20\left(\frac{1}{2}\right) + 85$

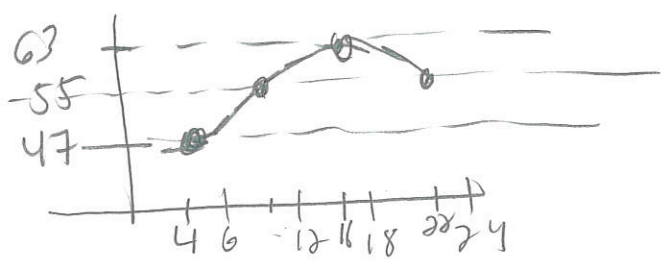
75

13. Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature varies between 47 and 63 degrees during the day and the average daily temperature first occurs at 10 AM. How many hours after midnight does the temperature first reach 51 degrees?

Amp = 8 $b = \frac{\pi}{12}$

$-8\cos\left(\frac{\pi}{12}(T-4)\right) + 55 = 51$

See other sheet.



15. A Ferris wheel is 20 meters in diameter and boarded from a platform that is 2 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 6 minutes. How many minutes of the ride are spent higher than 13 meters above the ground?

See other sheet.

33

6.5 Exercises

(1) $\tan A = \frac{8}{5}$

$\arctan\left(\frac{8}{5}\right) = m\angle A$ $\quad 57.99^\circ$

$m\angle B = 90 - 57.99$

$= 32^\circ$

$c = \sqrt{64+25}$

$\sqrt{89} \approx 9.4$

---

(2) $\tan A = \frac{3}{7}$ $\quad \arctan\left(\frac{3}{7}\right) = m\angle A = 23.2^\circ$

$m\angle B = 90 - 23.2 \approx 66.8^\circ$

$c = \sqrt{9+49}$

$\sqrt{58} \approx 7.6$

---

(3) $\sin A = \frac{7}{15}$ $\quad m\angle A = \arcsin\left(\frac{7}{15}\right)$

$\approx 27.8^\circ$

$m\angle B = 90 - 27.8$

$\approx 62.2^\circ$

$b = \sqrt{225-49} = \sqrt{176} \approx 13.3$

6.5 Solutions P2

(4) $\cos A = \frac{10}{12}$ $m\angle A = \arccos\left(\frac{5}{6}\right)$

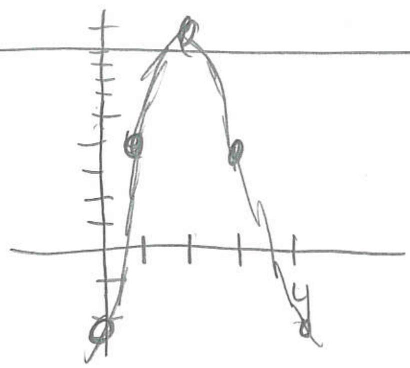
$\approx 33.6^\circ$

$m\angle B = 90 - 33.6 \approx 56.4^\circ$

$a = \sqrt{144 - 100} \rightarrow \sqrt{44} \approx 6.6$

---

(5)



Amp = 6

Period = 4 $b = \frac{2\pi}{4}$ $\frac{\pi}{2}$

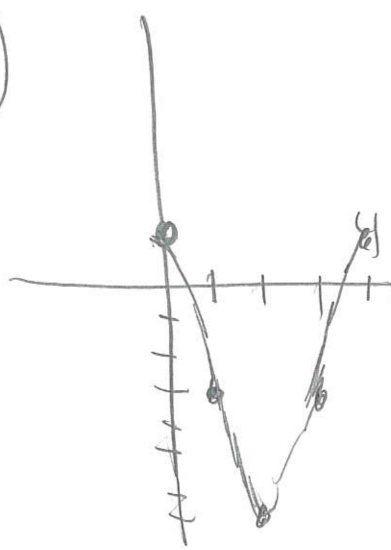
shifted right 2 for cosine

midline $y = 4$

$y = 6\cos\left(\frac{\pi}{2}(x-2)\right) + 4$

---

(6)



midline $y = -3$

amp = 4

per = 4 $b = \frac{\pi}{2}$

$4\cos\left(\frac{\pi}{2}x\right) - 3$

6.5 my work

#15

$$-8\cos\left(\frac{\pi}{12}(T-4)\right)+55=51$$

$$-8\cos\left(\frac{\pi}{12}(T-4)\right)=-4$$

$$\cos\left(\frac{\pi}{12}(T-4)\right)=\frac{1}{2}$$

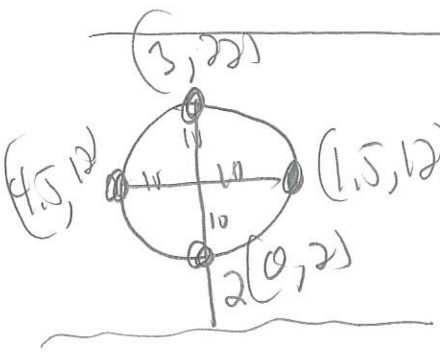
$$\cos u=\frac{1}{2}$$

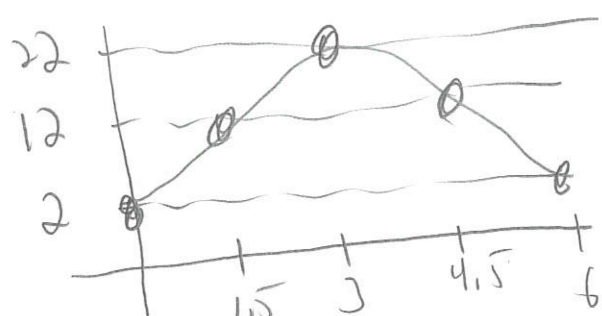
$$\frac{\pi}{12}(T-4)=\frac{\pi}{3}$$

$$T-4=\frac{\pi}{3}\cdot\frac{12}{1\pi}$$

$$T-4=4$$

$$\boxed{T=8}$$





$$h(T)=-10\cos\left(\frac{\pi}{3}(T)\right)+12$$

$$13=-10\cos\left(\frac{\pi}{3}T\right)+12$$

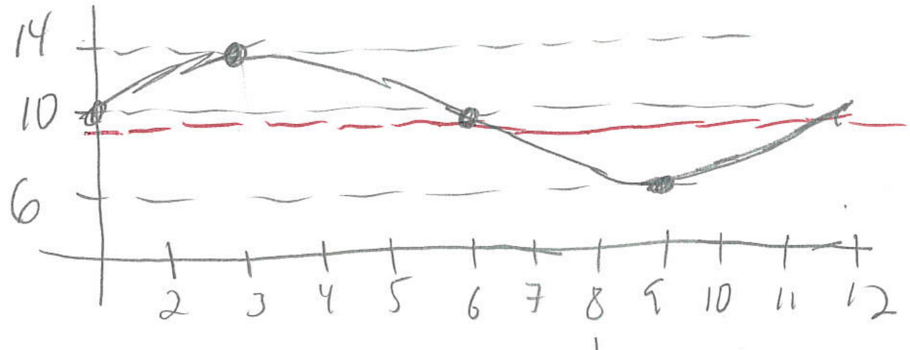
$$-\frac{1}{10}=\cos\left(\frac{\pi}{3}T\right)$$

$\frac{\pi}{3}T=1.671$ $T=1.6$

$\frac{\pi}{3}T=4.612$ $T=4.4$

$4.4-1.6=$ 2.8 mm

17. The sea ice area around the North Pole fluctuates between about 6 million square kilometers in September to 14 million square kilometers in March. Assuming sinusoidal fluctuation, during how many months are there less than 9 million square kilometers of sea ice?



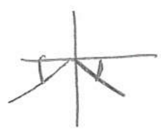
$4\sin\left(\frac{\pi}{6}T\right)+10=9$

$\sin\left(\frac{\pi}{6}T\right)=-\frac{1}{4}$

$\frac{\pi}{6}T=-0.253+2\pi k$

$T=-0.483+12$

$T=11.52$



$11.52-6.48$

5.03 months

$\frac{\pi}{6}T=3.39+2\pi k$

$T=6.483+12$

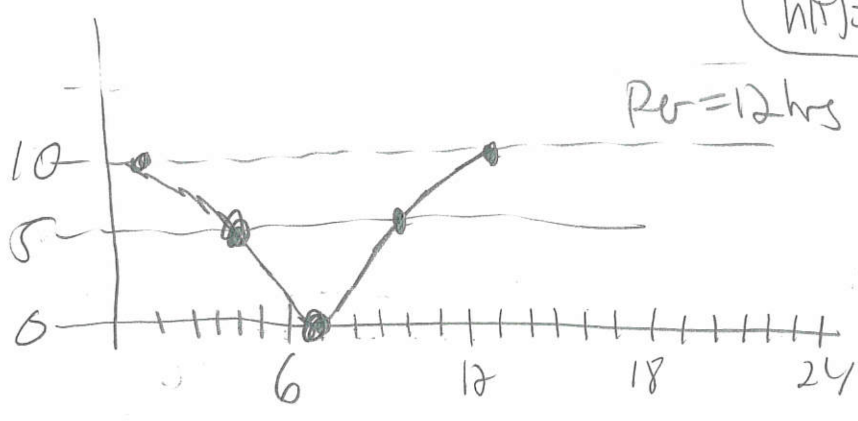
20. Suppose the high tide in Seattle occurs at 1:00 a.m. and 1:00 p.m, at which time the water is 10 feet above the height of low tide. Low tides occur 6 hours after high tides. Suppose there are two high tides and two low tides every day and the height of the tide varies sinusoidally. [UW]

a. Find a formula for the function $y=h(t)$ that computes the height of the tide above low tide at time t. (In other words, $y=0$ corresponds to low tide.)

b. What is the tide height at 11:00 a.m.?

$h(T)=5\cos\left(\frac{\pi}{6}(T-1)\right)+5$

Per $=12$ hrs



$5\cos\left(\frac{\pi}{6}(10)\right)+5$

$5\cos\left(\frac{5\pi}{3}\right)+5$

$5\left(\frac{1}{2}\right)+5$

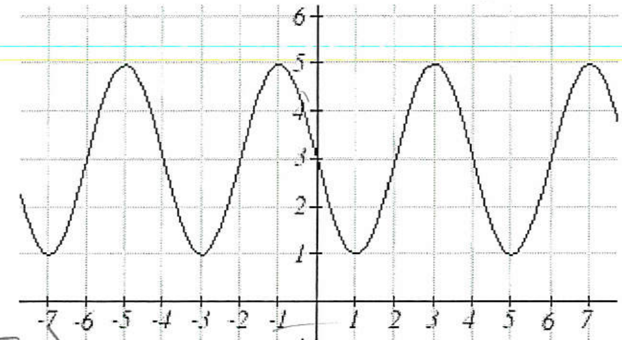
7.5 Ft

34

Practice Exam. See my work online

SHOW ALL WORK: Answers without adequate justification may not receive full credit. Give exact answers wherever possible. Given angle answers in radians unless otherwise specified.

1. (14pts) For graph to the right



a. Amplitude: 2

b. Midline: $y=3$

c. Period: 4

d. Equation using sine: $-2\sin\left(\frac{\pi}{2}t\right)+3$

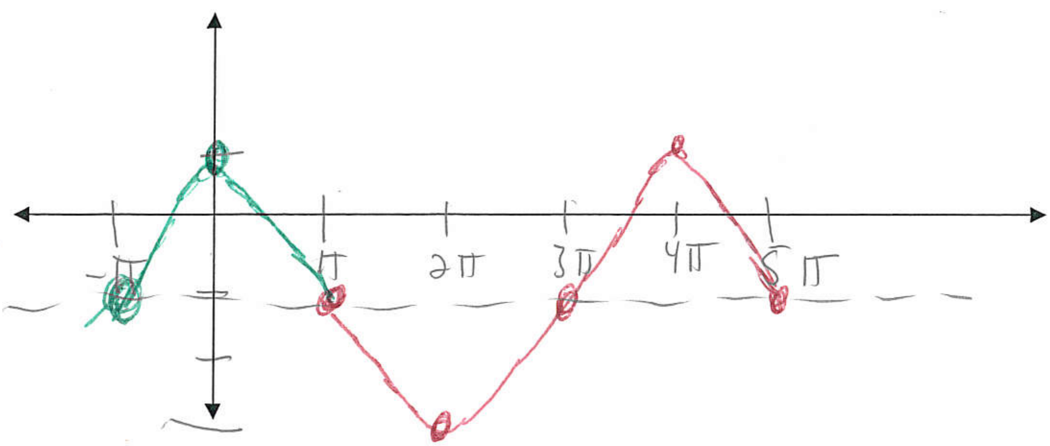
e. Equation using cosine: $2\cos\left(\frac{\pi}{2}(t+1)\right)+3$

2. (12pts) Given $f(t) = -2\sin\left(\frac{1}{2}(t-\pi)\right)-1$.

a. Amplitude = 2 b. Midline = $y=-1$

c. Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$ d. Horizontal Shift = Right π

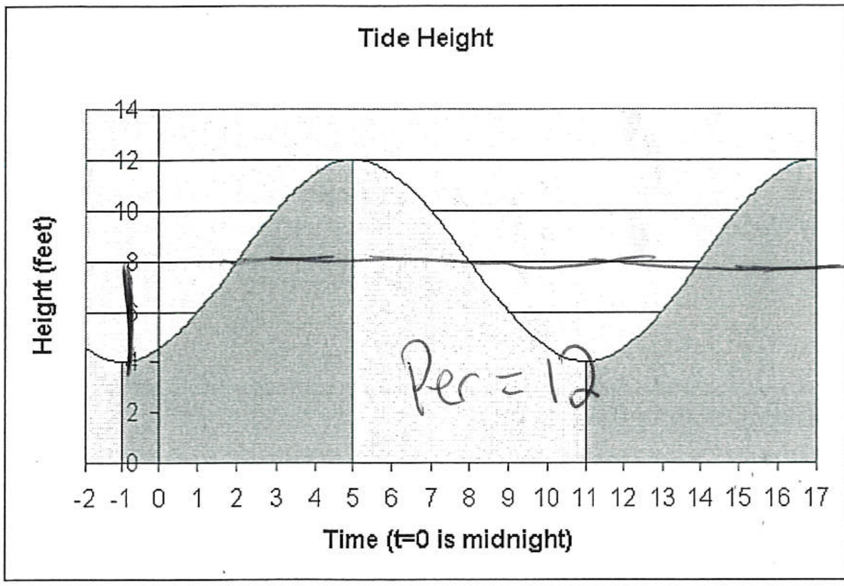
e. Sketch at least one full cycle of the graph. Be sure to label the axes.



3. (10pts) Below is part of a tide table. Find an equation for *h(t)*, the height of the tide, where *t* is measured in hours after midnight.

31

Practice Exam. See my work online



$y = -2\cos\left(\frac{\pi}{6}(T+1)\right) + 8$

4. (5pts) If $\tan(\theta) = A$, $\sin(\theta) = B$, and $\cos(\theta) = C$, then:

   a. $\sin(-\theta) =$ <u>$-B$</u>

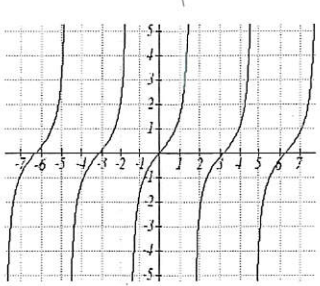
   b. $\tan(-\theta) =$ <u>$-A$</u>

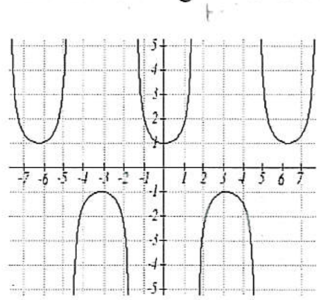
   c. $\sec(-\theta) =$ <u>C</u>

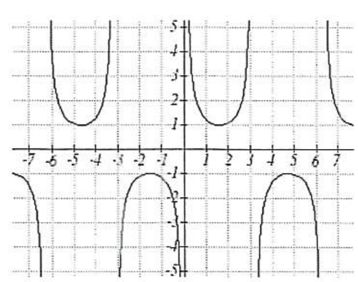
   d. $\csc(\theta + 2\pi) =$ <u>$\frac{1}{B}$</u>

   e. $\tan(\theta + \pi) =$ <u>A</u>

5. (3pts) Write the equation for the following BASIC trig functions *(they have not been transformed)*







Tan x | Sec x | Csc x

6. (4pts) Evaluate each of the following.

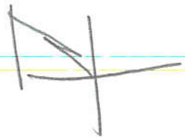
36

Practice Exam. See my work online

a) $\sin^{-1}\left(-\frac{1}{2}\right)$

$-\pi/6$

b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$



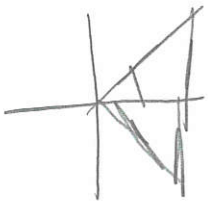
$3\pi/4$

7. (4pts) State the domain and range of $\sin^{-1}(x)$

D: $[-1, 1]$

R: $[-\pi/2, \pi/2]$

8. (4pts) Solve $\cos(\theta) = 0.2$ for all solutions $0 \le \theta < 2\pi$



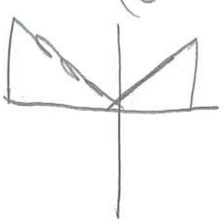
$\arccos(.2) = 1.369$

$2\pi - 1.369 = 4.914$

9. (8pts) Solve $2\sin\left(\frac{\pi}{3}t\right) = 1$ for the first three positive solutions.

$\frac{1}{2}, \frac{5}{2}, \frac{13}{2}$

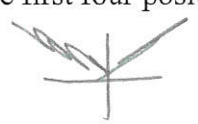
$\sin\left(\frac{\pi}{3}t\right) = \frac{1}{2}$



$\frac{\pi}{3}T = \frac{\pi}{6} + 2\pi k \rightarrow T = \frac{1}{2} + 6k$

$\frac{\pi}{3}T = \frac{5\pi}{6} + 2\pi k \quad T = \frac{5}{2} + 6\pi$

10. (10pts) Solve $3\sin(4t) = 1$ for the first four positive solutions.



$\sin 4T = \frac{1}{3}$

$\arcsin\left(\frac{1}{3}\right) = .34$

$\pi - .34 = 2.8$

$4T = .34 + 2\pi k$

$T = .085 + \frac{\pi}{2}k$

$4T = 2.8 + 2\pi k$

$T = 0.7 + \frac{\pi}{2}k$

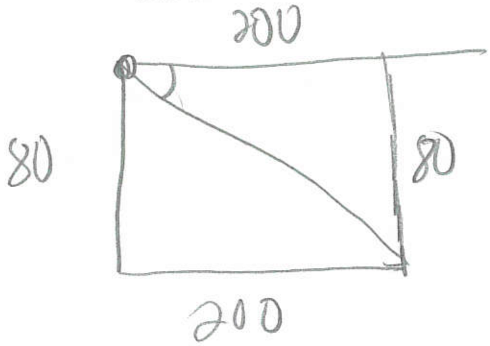
$.085, 0.7, 1.66, 2.27$

37

(7?)

Practice Exam. See my work online

11. (6pts) An airplane flies from Joint Base Lewis McChord (JBLM) to a undisclosed location 80 km south and 200 km east. In what direction should the plane fly?



The plane should fly ______ degrees south of east

$\arctan\left(\frac{80}{200}\right)$ 21.8°

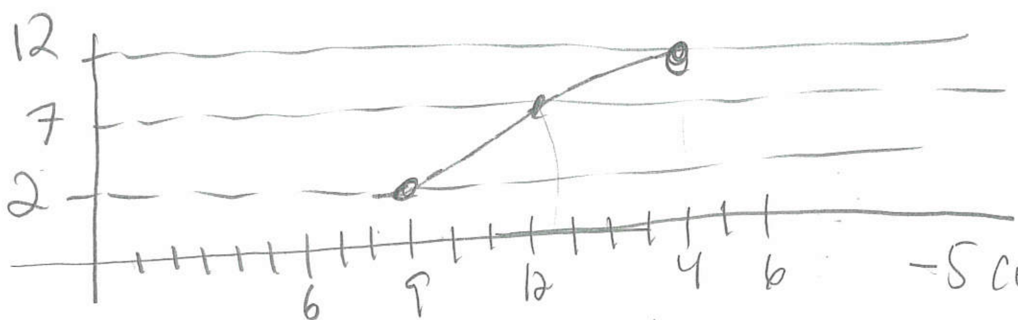
or

Azimuth 111.8°

Bearing

12. (17pts) A vacationer sits all day on the corner of a pier in Boston Harbor and notices that at 9am, when the water level is at its lowest, the waters depth is 2 feet. At 4 pm, the water has risen to its maximum depth of 12 feet. If the depth of the water level varies sinusoidally,

    a. Find a formula for the depth of the water as a function of time, t, since 9am.



$-5\cos\left(\frac{\pi}{7}(t-9)\right)+7$

Per = 14 hrs Amp = 5 midline y = 7

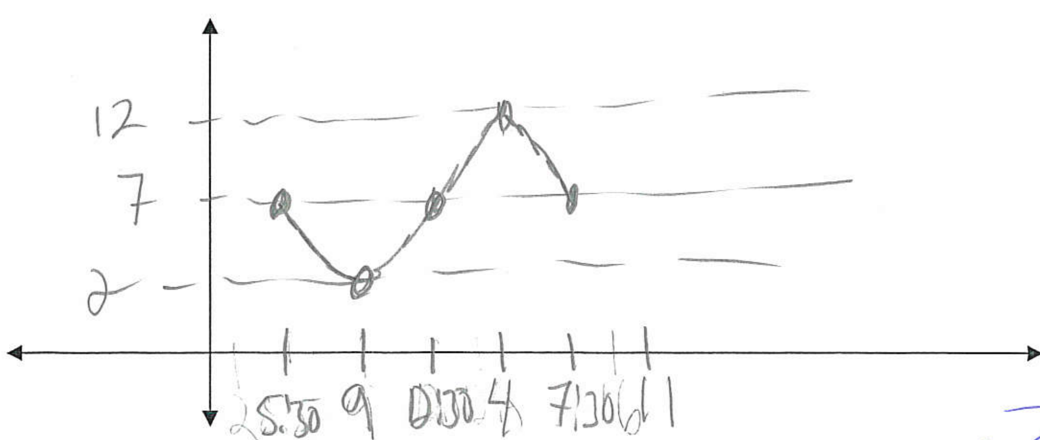
$-5\cos\left(\frac{\pi}{7}(10)\right)+7$

    b. What is the water depth at 7pm?

8.11 ft

7pm = 19 hours

    c. Sketch a graph of your function showing at least one full period



38