PS 1 Key

① $x^2+2y^2=11$

$x-y=-3 \rightarrow x=y-3$

$(y-3)^2+2y^2=11$ $\qquad (y-3)(y-3)\quad y^2-6y+9$

$y^2-6y+9+2y^2=11$

$3y^2-6y-2=0 \rightarrow$

$y=2.291 \qquad y=-0.291$

$x=2.293-3 \qquad y=-0.291-3$

$\boxed{(-0.707,\ 2.291)}$ and -3.291,

$\boxed{(-3.291,\ -0.291)}$

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② $\sin^2 x-\cos^2 x=0$

$1-\cos^2 x-\cos^2 x=0$ $\qquad \sin^2 x+\cos^2 x=1$

$1-2\cos^2 x=0$

$\cos^2 x=\frac{1}{2}$ $\qquad \boxed{\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}}$

$\cos x=-\frac{1}{\sqrt{2}} \qquad \cos x=\frac{1}{\sqrt{2}}$

$\cos x=-\frac{\sqrt{2}}{2} \qquad \cos x=\frac{\sqrt{2}}{2}$

(3) $e^{x} + e^{-x} = 5$

$e^{x} + \frac{1}{e^{x}} = 5$

$e^{2x} + 1 = 5e^{x}$

$e^{2x} - 5e^{x} + 1 = 0$

$e^{x} = 4.791$ $x = 1.567$

$e^{x} = 0.209$ $x = -1.565$

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(4) $\ln(5x-4) = \ln 4 - \ln(x-1)$

$\ln(5x-4) + \ln(x-1) = \ln 4$

$e^{\ln(5x-4)(x-1)} = e^{\ln 4}$

$5x^2 - 9x + 4 = 4$

$5x^2 - 9x = 0$

$x(5x-9) = 0$ However $\ln(5(0)-4)$ DNE

$x = \cancel{0}$ $x = \frac{9}{5}$ $\boxed{x = \frac{9}{5}}$

extraneous root

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(5) $3x^3 - 11x^2 \le 20x$

$3x^3 - 11x^2 - 20x \le 0$

$x(3x^2 - 11x - 20) \le 0$

$x(3x+4)(x-5) \le 0$ $\qquad x-5=0$

$x=0$

$x = -\frac{4}{3}$ $\qquad x=5$

$$\begin{array}{ccccccc} - & | & + & | & - & | & + \\ & -\frac{4}{3} & & 0 & & 5 & \end{array}$$

$$\boxed{(-\infty, -\tfrac{4}{3}] \cup [0, 5]}$$

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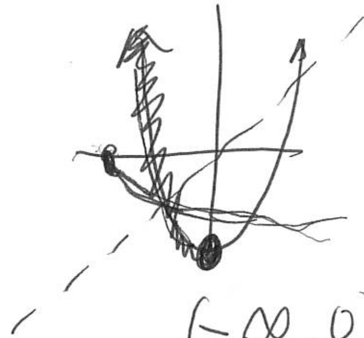
(6) $\sin(3(\frac{\pi}{6})) \to \sin\frac{\pi}{2} = 1$

$\sin(3(0)) \to \sin 0 = 0$

$\frac{1-0}{\frac{\pi}{6}-0} \qquad \frac{1}{\frac{\pi}{6}} = \boxed{\frac{6}{\pi}}$

7 $f(x) = x^2 - 9$



not 1:1

$x = y^2 - 9$

$x + 9 = y^2$

$\boxed{-\sqrt{x+9} = y}$

$(-\infty, 0)$ D

Range $(-9, \infty)$

Domain $[-9, \infty)$

Range $(-\infty, 0)$

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8 VA $x = -2$

$x = 5$

$\boxed{\dfrac{2x(x-2)}{(x+2)(x-5)}}$

Zeros

$x = 0$

$x = 2$

$HA = y = 2$

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9 $\lim_{x \to -\infty} f(x) = 2$

$\lim_{x \to \infty^+} f(x) = 2$

Left end behavior model

$y = 2$

(10) $y = x^3 + x^2 - 8x - 7 \text{ @ } x = 2$ $\dfrac{f(2+h) - f(2)}{2+h-2}$

$$\lim_{h \to 0} \frac{(2+h)^3 + (2+h)^2 - 8(2+h) - 7 - [8+4-16-7]}{2+h-2}$$

$12 - 16 - 7$

$-[-4-7]$

$(2+h)^3 = (2+h)(2+h)(2+h)$
$(2+h)(4+4h+h^2)$

$$\begin{array}{l} 8 + 8h + 2h^2 \\ \quad 4h + 4h^2 + h^3 \\ \hline 8 + 12h + 6h^2 + h^3 \end{array}$$

$(2+h)^2 = 4 + 4h + h^2$

$-8(2+h) = -16 - 8h$

$$\begin{array}{llll} 8 & +12h & +6h^2 & +h^3 \\ 4 & 4h & h^2 & \\ -16 & -8h & & \\ -7 & & & \\ +11 & & & \\ \hline 0 & 8h & +7h^2 & +h^3 \end{array}$$

$$\lim_{h \to 0} \frac{h^3 + 7h^2 + 8h}{h} \rightarrow \frac{h(h^2 + 7h + 8)}{h}$$

$$\lim_{h \to 0} h^2 + 7h + 8$$

$$= \boxed{8}$$