

c. $f(x) = x^4 + 2x^3 - 5x^2 - 14x - 14$, $-1+i$ is a root

d. $f(x) = x^4 - 4x^3 + 10x^2 - 8x + 16$, $2-2i$ is a root

Exercise

11. For each of the following rational functions, find the requested information and sketch. Calculator verify.

a. $y = \frac{3}{x+2}$

a. domain: $x \neq -2$

b. VA: $x = -2$

c. HA: $y = 0$ Bottom heavy

d. Zeros none

e. Interval Work

f. y -intercept: $(0, \frac{3}{2})$

b. $y = \frac{3x}{x+2}$

a. domain: $x \neq -2$

b. VA: $x = -2$

c. HA: $y = 3$

d. Zeros $(0, 0)$

e. Interval Work

f. y -intercept: $(0, 0)$

c. $y = \frac{x}{x^2 + 2x - 3}$ $\frac{x}{(x+3)(x-1)}$

a. domain: $x \neq -3, x \neq 1$

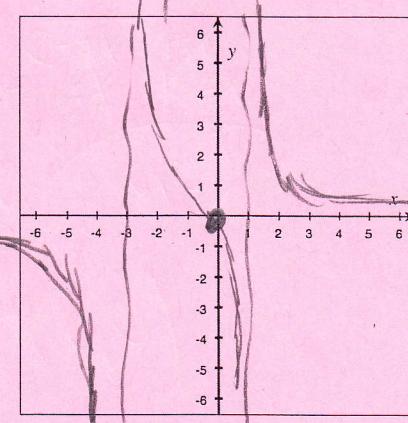
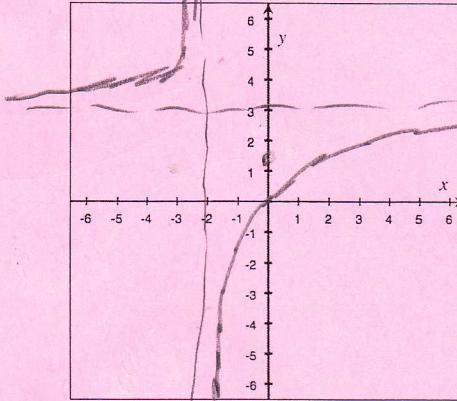
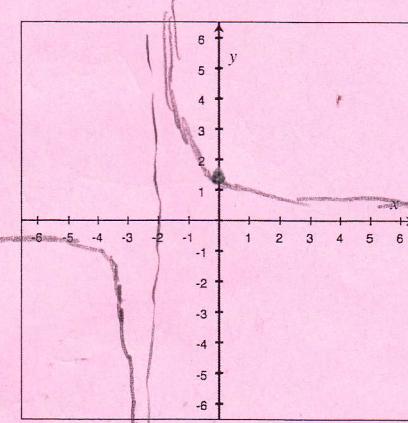
b. VA: $x = -3, x = 1$

c. HA: $y = 0$ Bottom heavy

d. Zeros $(0, 0)$

e. Interval Work

f. y -intercept: $(0, 0)$



$$(x+4)(x-4)$$

d. $y = \frac{x^2}{x^2 - 16}$

a. domain:

b. VA: $x = -4 \quad x = 4$

c. HA: $y = 1$

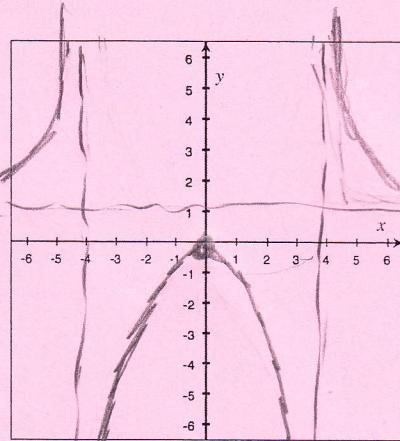
d. Zeros $(0, 0)$ Double zero

e. Interval Work

$$-4 \leq x \leq 4$$

f. y -intercept: $(0, 0)$

y is neg.



e. $y = \frac{x^2 - 3x - 4}{x^2}$

a. domain:

b. VA: $x \neq 0$ acts like $\frac{1}{x^2}$

c. HA: $y = 1$

d. Zeros $(x-4)(x+1)$

e. Interval Work

$$(4, 0) \quad (-1, 0)$$

f. y -intercept:

none

f. $y = \frac{x^2 - 2x + 1}{x^2 + 2x - 8}$

a. domain:

$$\frac{(x-1)(x-1)}{(x+4)(x-2)}$$

b. VA:

c. HA: $y = 1$

d. Zeros

$$x = -4 \quad x = 2$$

e. Interval Work

$$(1, 0) \text{ Dbl root}$$

f. y -intercept:

$$(0, -\frac{1}{8})$$

g. $y = \frac{-4x}{x^2 - x - 12}$

a. domain: $x \neq 4 \quad x \neq -3$

b. VA: $x = 4 \quad x = -3$

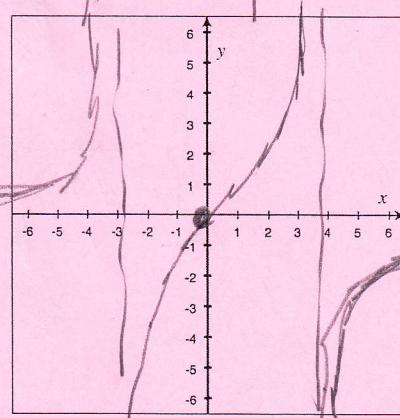
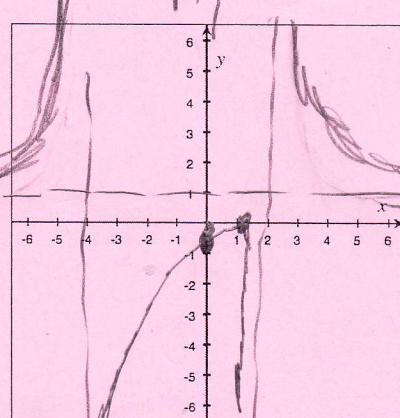
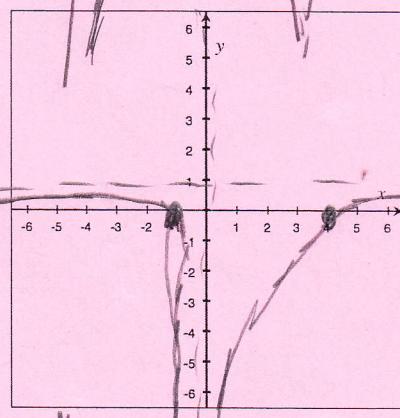
c. HA:

d. Zeros $(0, 0) \quad y = 0$

e. Interval Work

f. y -intercept:

$$(0, 0)$$



h. $y = \frac{4x+8}{x^2+4}$

a. domain: all reals

b. VA: none

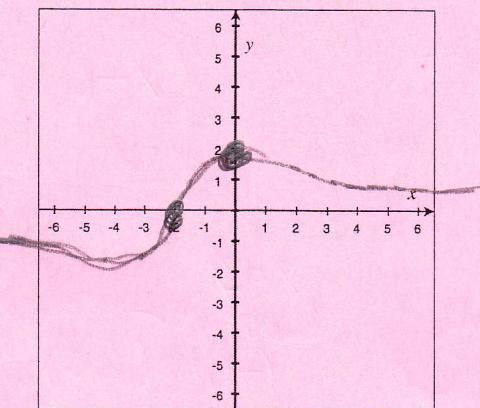
c. HA: $y = 0$

d. Zeros $\rightarrow x = (-2, 0)$

e. Interval Work

f. y -intercept:

$$y = (0, 2)$$



i. $y = \frac{x^2+4}{x^2+1}$

a. domain: All reals

b. VA: none

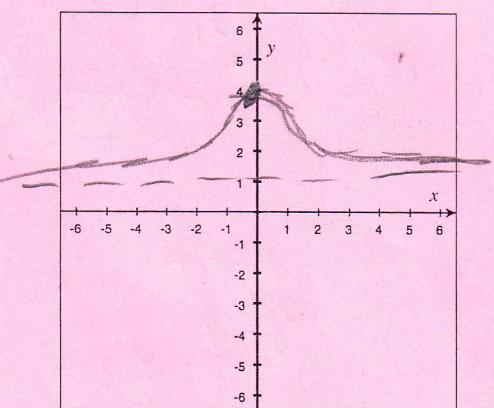
c. HA: $y = 1$

d. Zeros none

e. Interval Work

f. y -intercept:

$$(0, 4)$$



j. $y = \frac{1}{x} - \frac{x}{x+2}$

$$\frac{x+2}{x(x+2)} - \frac{x^2}{x(x+2)}$$

$$\frac{-x^2 + x + 2}{x(x+2)} = 0$$

$$\frac{x^2 - x - 2}{(x-2)(x+1)} = 0$$

a. domain: $x \neq 1$ $x \neq -2$

b. VA: $x = 0$ $x = -2$

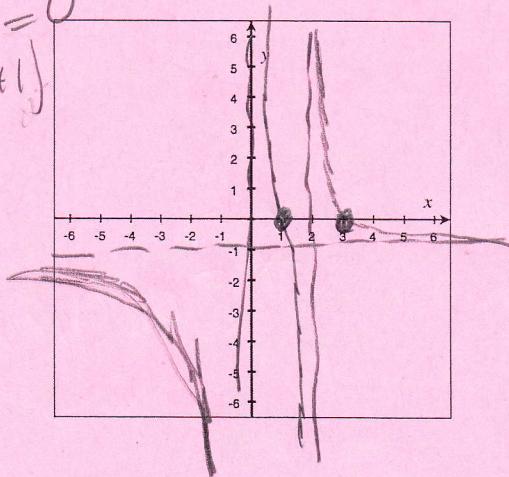
c. HA: $y = -1$

d. Zeros $x = 2$ $x = -1$

e. Interval Work

f. y -intercept:

$$\text{none}$$



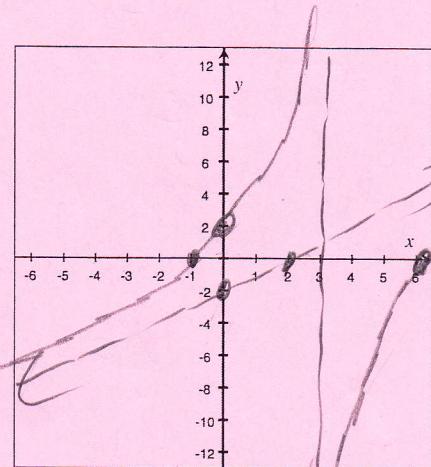
12. For each of the following rational functions, find the requested information and sketch. Verify using the calculator.

$$a. y = \frac{x^2 - 5x - 6}{x - 3}$$

Synthetic/Long Division:

$$\begin{array}{r} 3 \\[-0.5ex] | \overline{)1 \quad -5 \quad -6} \\ \underline{+3} \quad \quad \quad \\ 1 \quad -2 \quad -12 \\ \end{array}$$

$$x - 2 + \frac{-12}{x - 3}$$



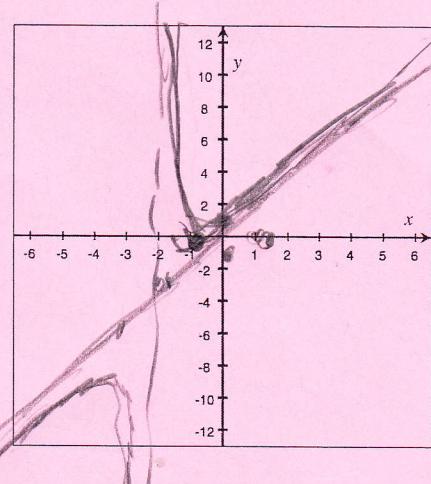
f. y-intercept:

$$(0, 2)$$

$$b. y = \frac{x^2 + 2x + 1}{x + 2}$$

Synthetic/Long Division:

$$\begin{array}{r} -2 \\[-0.5ex] | \overline{)1 \quad 2 \quad 1} \\ \underline{+2} \quad \quad \quad \\ 1 \quad 0 \quad 1 \end{array}$$



f. y-intercept:

$$(0, \frac{1}{2})$$

$$c. y = \frac{x - x^2}{x + 1}$$

Synthetic/Long Division:

$$-x(x - 1)$$

a. domain: $x \neq -1$

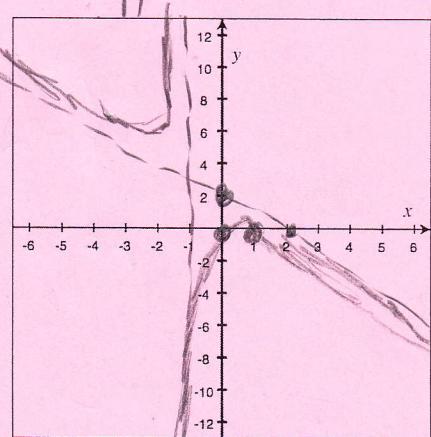
b. VA: $x = -1$

c. OA: $y = -x + 2$

d. Zeros: $(0, 0), (1, 0)$

e. Interval Work:

$$\begin{array}{r} -1 \\[-0.5ex] | \overline{-1 \quad 1 \quad 0} \\ \underline{+1} \quad \quad \quad \\ -1 \quad 2 \quad -2 \end{array}$$



f. y-intercept:

$$-x + 2 + \frac{-2}{x + 1}$$

d. $y = \frac{x^3}{x-1}$

Synthetic/Long Division:

$$\begin{array}{r} 111000 \\ \downarrow 111 \\ 1111 \end{array}$$

$$x^2 + x + 1 + \frac{1}{x-1}$$

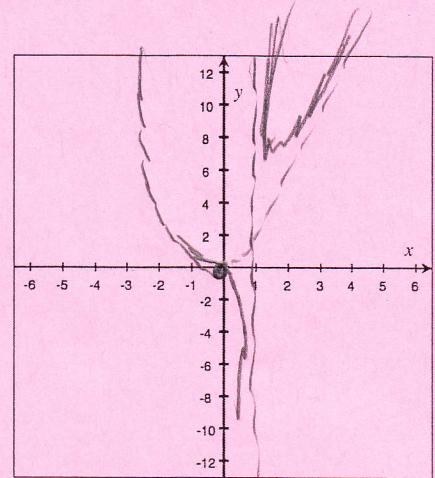
a. domain: $x \neq 1$

b. VA: $x = 1$

c. OA: $x^2 + x + 1$

d. Zeros: 0 (No real zeros)

e. Interval Work:



f. y-intercept:

$$(0, 0)$$

e. $y = \frac{x^3 - 2x^2 - 4x + 8}{x-1}$

Synthetic/Long Division:

$$\begin{array}{r} 111-2-48 \\ \downarrow 111 \\ 1-1-53 \end{array}$$

a. domain:

b. VA: $x = 1$

c. OA: $x^2 - x - 5$

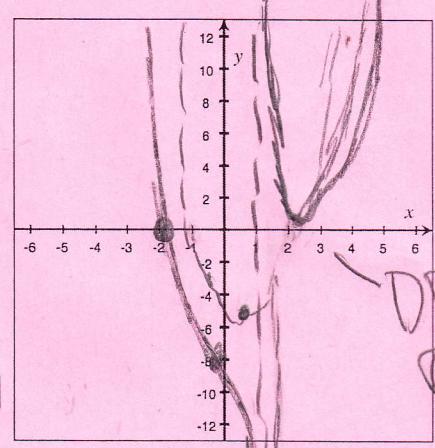
d. Zeros: $(-2, 0), (2, 0)$ DBL

e. Interval Work:

f. y-intercept:

$$(0, -8)$$

$$x^2 - x - 5 + \frac{3}{x-1}$$



f. $y = \frac{x^3 - 3x + 2}{x^2 - 2x}$

Synthetic/Long Division:

$$\begin{array}{r} \frac{1}{4} - \frac{1}{2} - \frac{15}{2} \\ \frac{3}{4} - \frac{20}{4} \end{array}$$

$$\frac{1}{2} - \frac{21}{4}$$

$$\left(\frac{1}{2}, -\frac{5}{4}\right)$$

a. domain: $x \neq 0 \text{ and } x \neq 2$

b. VA: $= 0$

c. OA: $y = x + 2$

d. Zeros: $-1, 2$

e. Interval Work:

f. y-intercept: $NO \ y = -1$

$Hole @ x = 2$

$$\begin{array}{r} x+2 \\ x^2-2x \quad | \quad x^3+0x^2-3x+2 \\ \underline{-x^3+2x^2} \\ \underline{2x^2-3x+2} \\ \underline{-2x^2+4x} \\ x+2 \end{array}$$

