AP Calculus
2.1 Worksheet Day 1

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The only way to guarantee the existence of a limit is to algebraically prove it. Describe the different ways you can <u>investigate</u> the existence of a limit.

Graphically, Tabular, Seeing what happens from either side.

2. Using words, explain what is meant by the expression $\lim_{q \to c} f(q) = T$.

as q approaches "c" from both sides, the function approaches 'T'

3. How do you find the average speed of an object?

Find the "slope" between two times

4. Suppose an object moves along the x-axis with it's position function given by $x(t) = 5t^2 + 7t$, where t is measured in seconds.

a) What is the average speed from $t = 2$ to $t = 4$ seconds?

$x(2) = 20 + 14 \to \cancel{48}\ 34$

$x(4) = 80 + 28 \to 108$

b) How fast is the object moving at exactly $t = 4$ seconds?

$\lim_{h \to 0} \frac{5(4+h)^2 + 7(4+h) - [108]}{h}$ $\quad \frac{5(16 + 8h + h^2) + 28 + 7h - 108}{h}$ $\quad \frac{108 - 34}{4 - 2} \to 37$ units/sec

$\frac{80 + 40h + 5h^2 + 28 + 7h - 108}{h}$ $\quad \frac{47h + 5h^2}{h} \to \lim_{h \to 0} 47 + 5h = 47$ units/sec

5. An rover on another planet drops an object off a cliff. The object falls $y = gt^2$ m in t sec, where g is a constant. Five seconds after the object was dropped it lands 30 m below.

a) Find the value of g. $\quad 30 = g(5)^2 \to g = \frac{6}{5}$

b) Find the average speed for the fall.

$y(0) = 30 \quad y(5) = 0 \quad \frac{30 - 0}{0 - 5} = |-6| = 6$ m/sec

c) With what speed did the rock hit the bottom?

lim

See next page

6. Assume $\lim_{x \to b} f(x) = 7$ and $\lim_{x \to b} g(x) = -3$.

a) $\lim_{x \to b}(f(x) + g(x)) =$

$\lim f(x) + \lim g(x)$
$7 + -3$ $\quad 4$

b) $\lim_{x \to b}(f(x) \cdot g(x)) =$ $\quad 7(-3) = -21$

c) $\lim_{x \to b} 4g(x) =$

$4(7) = 28$

d) $\lim_{x \to b}\left(\frac{f(x)}{g(x)}\right) =$ $\quad \frac{7}{-3}$

7. When asked to evaluate the limit of a function, what should be done first? Substitute in the value to see that you don't have $\frac{0}{0}$

8. Evaluate the following limits by using direct substitution.

a) $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$ $\sec\frac{7\pi}{6} = \frac{1}{\cos\frac{7\pi}{6}}$ $-\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$

b) $\lim_{x\to 4} \sqrt[3]{x+4}$ $\sqrt[3]{8} = 2$

c) $\lim_{x\to \frac{1}{2}} 3x^2(2x-1)$ $3(\frac{1}{4})(1-1) = 0$

d) $\lim_{y\to 2} \frac{y^2+5y+6}{y+2}$ $\frac{4+10+6}{4} = \frac{20}{4} \to 5$

e) $\lim_{x\to -2} (x-6)^{\frac{2}{3}}$ $\sqrt[3]{-2-6}^{\,2} \to (\sqrt[3]{-8})^2 \to (-2)^2 = 4$

f) $\lim_{x\to 2} \sqrt{x+3}$ $\sqrt{5}$

9. Explain why you cannot use direct substitution to determine each of the following limits.

a) $\lim_{x\to -2} \sqrt{x-2}$

$\sqrt{-2-2} = \sqrt{-4}$

-2 not in domain of $\sqrt{x-2}$

b) $\lim_{x\to 0} \frac{1}{x^2}$

Can't divide by 0

c) $\lim_{x\to 0} \frac{(4+x)^2-16}{x}$ $\frac{16-16}{0}$ $\frac{0}{0}$

10. If a limit does not exist, there are 3 possible reasons why. List all three possible reasons why a limit may not exist.

Infinite limit. Jump discontinuity. ~~Function not de~~ Ossilaing disc.

11. Find each limit, or explain why the limit does not exist.

a) $\lim_{x\to 2} f(x)$, if $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln x & \text{for } 2 < x \le 4 \end{cases}$

$\ln 2$ lim on left $\neq$ lim on rt

$4\ln 2$

b) $\lim_{x\to 2^+} f(x)$, if $f(x) = \begin{cases} 3x+1 & , x < 2 \\ \frac{5}{x+1} & , x \ge 2 \end{cases}$

2 from rt $\frac{5}{3}$ ✓

c) $\lim_{x\to 1} \frac{x^2-4}{x-1}$ Infinite limit

$\frac{1-4}{1-1}$ $\frac{-3}{0}$

d) $\lim_{x\to 2} \frac{x+1}{x^2-4}$ $\frac{3}{0}$ infinite limit.

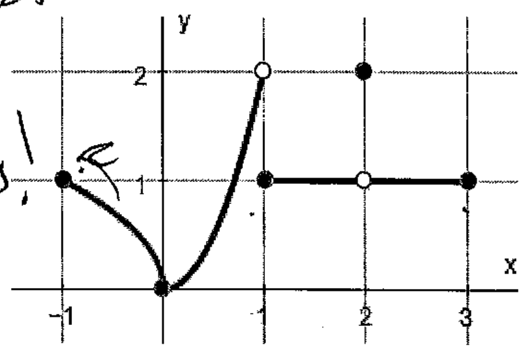
12. Determine whether each statement about the graph below is True or False.

F Left R5 Same

a) $\lim_{x \to -1^+} f(x) = 1$ ✓

b) $\lim_{x \to 2} f(x)$ does not exist

~~Jump.~~



F

c) $\lim_{x \to 2} f(x) = 2$ 4

~~Left~~

d) $\lim_{x \to 1^-} f(x) = 2$

From Left Yes!

e) $\lim_{x \to 1^+} f(x) = 1$ ✓

1 from Rt ✓

f) $\lim_{x \to 1} f(x)$ does not exist

Yes, Jump.

g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

Yes

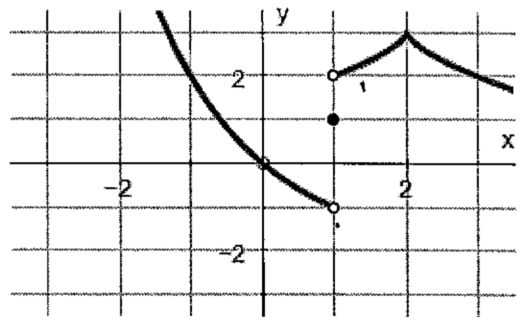
h) $\lim_{x \to c} f(x)$ exists at every c in (–1, 1)

Yes.

i) $\lim_{x \to c} f(x)$ exists at every c in (1, 3)

yes

13. Use the graph of $f(x)$ to estimate the limits and value of the function, or explain why the limit does not exist.



a) $\lim_{x \to 1^+} f(x)$ = 2

b) $\lim_{x \to 1^-} f(x)$ –1

c) $\lim_{x \to 1} f(x)$ Jump

d) f (1) 1

e) $\lim_{x \to 2^+} f(x)$ 3

f) $\lim_{x \to 2^-} f(x)$ = 3

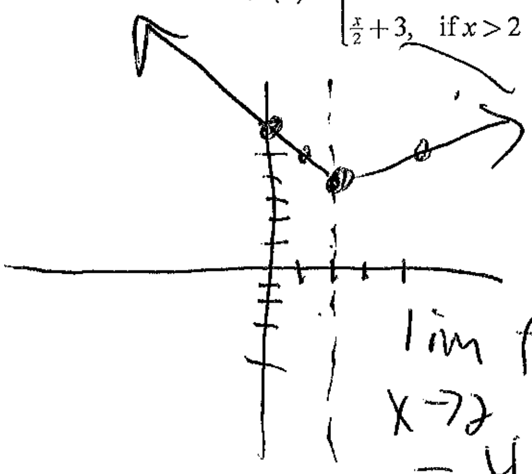
g) $\lim_{x \to 2} f(x)$ Yes 3

h) f (2) 3

14. For each of the following functions, (i) draw the graph, (ii) determine $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$, and (iii) explain what the value of $\lim_{x \to c} f(x)$ is or explain why it doesn't exist.

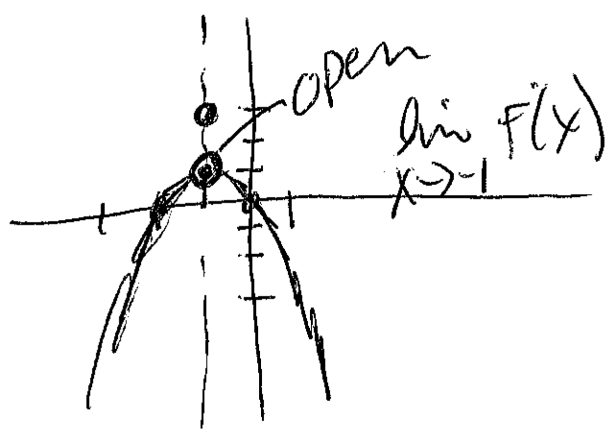
a) $c = 2, \ f(x) = \begin{cases} 6 - x, & \text{if } x < 2 \\ 4, & \text{if } x = 2 \\ \frac{x}{2} + 3, & \text{if } x > 2 \end{cases}$

b) $c = -1, \ f(x) = \begin{cases} 1 - x^2, & \text{if } x \neq -1 \\ 3, & \text{if } x = -1 \end{cases}$



$\frac{1}{2}x + 3$

$\lim_{x \to 2} f(x) = 4$

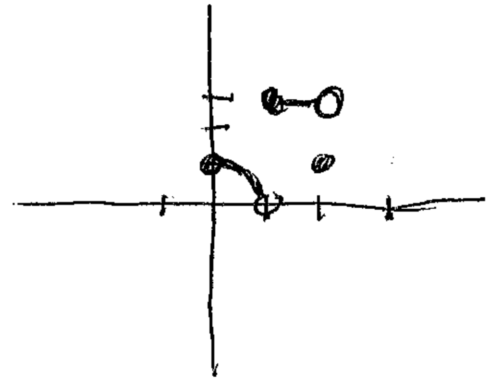


open

$\lim_{x \to -1} f(x) = 1$

a semi circle with radius 1 centred @ (0,0)

15. Suppose $f(x)=\begin{cases}\sqrt{1-x^2}, & \text{if } 0\le x<1\\ 3, & \text{if } 1\le x<2\\ 1, & \text{if } x=2\end{cases}$. Draw a graph of $f(x)$, then answer the following questions.



a) At what points c in the domain of f does $\lim_{x\to c} f(x)$ exist?

$[0,1)\cup(1,2)$ note the open intervals

b) At what point(s) c does only the left-hand limit exist?

$(0,1]$

c) At what point(s) c does only the right-hand limit exist?

$[0,1)\cup[1,2)$

16. A water balloon dropped from the roof of a small building falls $y=4.9t^2$ m in t sec. Suppose you wanted to know the speed of the water balloon at exactly $t=2$ seconds. Originally, we used values of t really close to 2 and found the average rate of change between them. Let's try something a little different …

a) Instead of using a numeric value "close" to 2, what would be the average speed of the balloon between $t=2$ and $t=2+h$? (Simplify the expression as much as you can)

$\frac{4.9(2+h)^2-4.9(2)^2}{h}$ $\quad \frac{4.9(4+4h+h^2)-19.6}{h}$

$\frac{19.6+19.6h+4.9h^2-19.6}{h}$ $\quad 19.6+4.9h$

b) To find the speed of the balloon at $t=2$, it is tempting to simply plug in $h=0$, however, this yields $\frac{0}{0}$, which is an "indeterminate form". We CAN however, evaluate your simplified expression from part *a* using limit as $h\to 0$. Evaluate this limit.

$\lim_{h\to 0} 19.6+4.9h = 19.6$ m/sec

c) Now find and compare the speed of the balloon at $t=2$ like you did earlier in question #4.

AP Calculus
2.1 Worksheet Day 2

$\lim_{x \to a} f(x) \to \infty$

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. When evaluating limits, what does it mean if direct substitution gives you $\frac{?}{0}$? Infinite limit

   a VA

2. When evaluating limits, what does it mean if direct substitution gives you $\frac{0}{0}$?

   There is a hole in the graph

3. What are the methods (options) for dealing with the result $\frac{0}{0}$?

   remove the discontinuity

4. Evaluate the following limits algebraically.

a) $\lim_{x \to 3} \frac{x^2 - x - 6}{x-3}$ $\quad \frac{(x-3)(x+2)}{x-3}$

$\lim_{x \to 3} x+2 \quad \boxed{5}$

b) $\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$ $\quad \frac{2-(2+x)}{2(2+x)} \cdot \frac{1}{x} \quad \frac{-1}{2(2+x)}$

$\lim_{x \to 0} \frac{-1}{2(2+0)} = -\frac{1}{4}$

c) $\lim_{x \to 0} \frac{\sqrt{2x+1} - 1}{x}$ $\quad \frac{(\sqrt{2x+1}+1)}{(\sqrt{2x+1}+1)}$

$\frac{2x+1-1}{x(\sqrt{2x+1}+1)} = \lim_{x \to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{2x+1}+1)} \cdot \frac{2}{\sqrt{1}+1} = \frac{2}{2} = 1$

d) $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x-4}$ See next page

e) $\lim_{x \to 1} \frac{x-1}{x^2-1}$ $\quad \frac{x-1}{(x-1)(x+1)}$

$\lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

f) $\lim_{x \to 0} \frac{(4+x)^2 - 16}{x}$ $\quad \frac{(4+x)^2+16}{(4+x)^2+16}$

$\cancel{\frac{(4+x)^4 - 256}{x[(4+x)^2+16]}}$ next B

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

g) $\lim_{t \to 2} \frac{t^2 - 3t + 2}{t^2 - 4}$

h) $\lim_{x \to 0} \frac{(2+x)^3 - 8}{x}$ Back

$\frac{(T-2)(T-1)}{(T+2)(T-2)}$ $\quad \lim_{T \to 2} \frac{T-1}{T+2} \quad \frac{1}{4}$ $\quad x^4 +$

One of the limits you should know is $\lim_{x\to 0}\frac{\sin x}{x} = \underline{\ 1\ }$. This limit ONLY works when the denominator matches the inside of the sine function. If they do not match, you cannot change the inside of a sine function without a trig identity. Your goal will be to correctly show the algebra in order to use this limit.

5. Evaluate each of the following limits analytically. *Be sure to show ALL steps in your evaluation.*

a) $\lim_{x\to 0}\frac{\sin x}{5x}$

$\lim_{x\to 0} \frac{1}{5}\cdot\frac{\sin x}{x}$

$\lim_{x\to 0}\frac{1}{5}\cdot\lim_{x\to 0}\frac{\sin x}{x} = \frac{1}{5}$

b) $\lim_{x\to 0}\frac{\sin 5x}{x}$

$\lim_{x\to 0} 5\frac{\sin x}{x}$ $\quad \lim_{x\to 0} 5\cdot\lim_{x\to 0}\frac{\sin x}{5}$

5

c) $\lim_{x\to \pi/4}\frac{\sin(x-\pi/4)}{x-\pi/4}$

$\frac{\sin 0}{0} = 1$

d) $\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x}$

$\frac{3\cdot 4}{3}$

4

6. Evaluate each of the following by combining properties of limits and your algebra skills.

a) $\lim_{x\to 0}\frac{x+\sin x}{x}$

$\lim_{x\to 0}\frac{x}{x} + \lim_{x\to 0}\frac{\sin x}{x}$

$1 + 1$

2

b) $\lim_{x\to 0}\frac{\tan x}{x}$

$\frac{\sin x}{\cos x} \div x$ $\quad \frac{\sin x}{\cos x}\cdot\frac{1}{x}$

$\lim_{x\to 0}\frac{\sin x}{x}\lim_{x\to 0}\frac{1}{\cos x}$

$1\cdot 1 = 1$

c) $\lim_{x\to 0}\frac{\sin x}{2x^2-x}$

$\frac{\sin x}{x}\cdot\frac{1}{2x-1}$

$1\cdot -1 = -1$

d) $\lim_{x\to 0}\frac{\sin^2 x}{x}$

$\frac{\sin x}{x}\cdot \sin x$

$1\cdot 0 = 0$

7. Consider $\lim_{x\to 0}\frac{f(x)-f(0)}{x^2} =$

a) If you use direct substitution, what result do you get?

$\frac{f(0)-f(0)}{0}$ $\quad \frac{0}{0}$

b) Evaluate the limit if $f(x)=2x^2+1$.

$\lim_{x\to 0}\frac{2x^2+1-1}{x^2} = \frac{2x^2}{x^2} = 2$

8. If $a \neq 0$, then $\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4} =$

$\frac{(x+a)(x-a)}{(x^2+a^2)(x+a)(x-a)}$ $\quad \frac{1}{x^2+a^2}$ $\quad \frac{1}{2a^2}$

9. Evaluate the following limits analytically (all mixed up):

a) $\lim_{x\to 0}\frac{\frac{3}{4+x}-\frac{3}{4}}{x}$

$\frac{12-3(4+x)}{4(4+x)}\cdot\frac{1}{x}$

$\lim_{x\to 0}\frac{-3}{4(4+x)}=-\frac{3}{16}$

b) $\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}$

$\frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)}$

$\lim_{x\to 0}\frac{5x+8}{3x^2-16}=-\frac{1}{2}$

c) $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4}$

$\frac{\sqrt{3+1}}{3-4}\quad\frac{\sqrt{4}}{-1}\quad\frac{2}{-1}$

d) $\lim_{x\to 0}\frac{x^2-3x}{x}$

$\frac{x(x-3)}{x}\quad\lim_{x\to 0}x-3=-3$

e) $\lim_{x\to 1}\frac{x}{x^2-x}$

$\frac{x}{x(x-1)}\quad\lim_{x\to 1}\quad\frac{1}{0}$ - infinite DNE

f) $\lim_{x\to 0}\frac{\sin 2x}{x}$

2

g) $\lim_{x\to 0}\frac{\sin 7x}{3x}$

$\frac{7}{3}$

h) $\lim_{x\to 4}\frac{x^2-5x+4}{x^2-2x-8}$

$\frac{\cancel{(x-4)}(x-1)}{\cancel{(x-4)}(x+2)}$

$\lim_{x\to 4}\frac{x-1}{x+2}\quad\frac{3}{6}\quad\frac{1}{2}$

12. Evaluate $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$.

♫: h is going to 0 … not x … so treat this as if h is the variable … your final answer will have a x in it.

$\frac{x^2+2h+h^2-x^2}{h}\quad\lim_{h\to 0}2+h=2$

13. Suppose $g(x)=\begin{cases}2-x, & \text{if } x\le 1\\ \frac{x}{2}+1, & \text{if } x>1\end{cases}$.

a) $\lim_{x\to 1^-}g(x)=$ 1

b) $\lim_{x\to 1^+}g(x)=$ $\frac{3}{2}$

c) $\lim_{x\to 1}g(x)=$ DNE

d) $g(1)=$ 1

Still working on this one

AP Calculus
2.2 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Answer the following questions:

   a) How do you find horizontal asymptotes? $\lim_{x \to \infty}$

   b) Which of the parent functions have horizontal asymptotes? List the function(s) and asymptote(s)

   $\frac{1}{x}$ $y=0$ $\frac{1}{x^2}$ $y=0$ e^x $y=0$

   c) How do you find vertical asymptotes?

   Division by 0 or not in domain like $\log x$

   d) Which of the parent functions have vertical asymptotes? List the function(s) and asymptote(s)

   $\frac{1}{x}$ $\frac{1}{x^2}$ $\log x$ $\ln x$ $x=0$

   e) When must you look for oblique (slanted) asymptotes? How do you find them?

   Rational Function long Division / Synthetic Division

2. For each of the following, find (i) $\lim_{x\to\infty} f(x)$ and (ii) $\lim_{x\to-\infty} f(x)$. Then (iii) identify all horizontal asymptotes, if any.

   a) $f(x)=\frac{x-2}{2x^2+3x-5}$

   $y=0$

   b) $f(x)=\frac{4x^3-2x+1}{x^2-2x+1}$

   ~~y=4x~~ over

   $x^2 \cancel{-2x+1}\overline{)4x^3-2x+}$

   c) $f(x)=\frac{3x^2-x+5}{x^2-4}$

   $y=3$

   d) $f(x)=\frac{e^{-x}}{x}$ $\frac{1}{e^x} \cdot \frac{1}{x}$

   $\frac{1}{xe^x}$ $x\to\infty$ $y=0$

   $x\to-\infty$ $y\to 0$ $y\to 0$

   e) $f(x)=\frac{|x|}{x}$ Right $y=1$

   left $y=-1$

   f) $f(x)=\frac{\sin x}{2x^2+x}$ $\frac{\sin x}{x}$ $\frac{1}{2x+1}$

   $\lim_{x\to-\infty}$ or $x\to\infty$ 0 so 0

3. One of the functions in 2a – 2c has a slanted (oblique) asymptote. Explain why, and then find the asymptote.

   b) See brn

4. For each of the following, (i) find the vertical asymptotes of the graph of $f(x)$ and (ii) describe the behavior of the graph of $f(x)$ to the left and right of each asymptote.

$x=-2$ and $x=2$

$x=3$

a) $f(x)=\dfrac{1}{x-3}$

$\lim_{x\to 3^-} f(x)\to-\infty$ $\lim_{x\to 3^+} f(x)\to\infty$

b) $f(x)=\dfrac{1}{x^2-4}$

$\dfrac{1}{(x+2)(x-2)}$

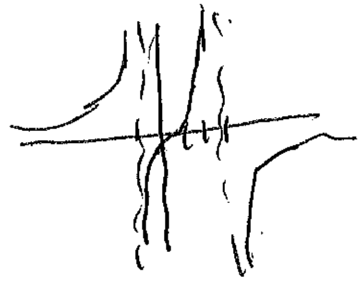
$\lim_{x\to -2^-} \infty$

$x\to -2^+$ $-\infty$

$-\infty$

c) $f(x)=\dfrac{1-x}{2x^2-5x-3}$

$\dfrac{-(x-1)}{(2x+1)(x-3)}$



$\lim_{x\to -\frac{1}{2}^-} \infty$

$\lim_{x\to -\frac{1}{2}^+} -\infty$

$\lim_{x\to 3^-} \infty$

$\lim_{x\to 3^+} -\infty$

5. Find the limit of $g(x)$ as (i) $x\to\infty$, (ii) $x\to-\infty$, (iii) $x\to 0^-$, and (iv) $x\to 0^+$

a) $g(x)=\begin{cases}\frac{1}{x} & \text{if } x<0\\ \frac{2x-3}{x+1} & \text{if } x\geq 0\end{cases}$

b) $g(x)=\begin{cases}\frac{3x}{x+1} & \text{if } x\leq 0\\ \frac{1}{x^2} & \text{if } x>0\end{cases}$

6. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim_{x\to 1} f(x)=2$ $\lim_{x\to 5^-} f(x)=\infty$

$\lim_{x\to 5^+} f(x)=\infty$ $\lim_{x\to\infty} f(x)=-1$

$\lim_{x\to-\infty} f(x)=0$ $\lim_{x\to-2^-} f(x)=\infty$

$\lim_{x\to-2^+} f(x)=-\infty$

7. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim_{x\to 2} f(x)=-1$ $\lim_{x\to 4^+} f(x)=-\infty$

$\lim_{x\to 4^-} f(x)=\infty$ $\lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to-\infty} f(x)=2$

8. Explain why there is no value L for which $\lim_{x\to\infty}\sin x=L$.

9. Let $f(x)=\dfrac{\cos x}{x}$.

a) Find the domain and range of f.

b) Is f even, odd, or neither? Justify your response.

c) Find $\lim\limits_{x\to\infty} f(x)$. Give a reason for your answer.

10. If k is a positive integer, then $\lim\limits_{x\to\infty} \dfrac{x^k}{e^x} = ?$ Explain your answer.
[Try letting $k = 2$ … what about $k = 10$? … what about $k = 1000$?]

11. ***Investigate*** using tables and graphs to determine the value of each limit: $\lim\limits_{x\to\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$ and $\lim\limits_{x\to-\infty} \dfrac{3x-2}{\sqrt{2x^2+1}}$

12. Evaluate each of the following limits using all methods learned from this chapter.

a) $\lim\limits_{x\to\infty}\left(\dfrac{2}{x}+1\right)\left(\dfrac{5x^2-1}{x^2}\right)$

b) $\lim\limits_{n\to\infty}\dfrac{3n^3-5n}{n^3-2n^2+1}=$

c) $\lim\limits_{x\to\infty}\left(5-\dfrac{2}{x^2}\right)=$

d) $\lim\limits_{x\to\pi/2} \sec x$

e) $\lim\limits_{x\to\infty} e^{-x}\cos x$

f) $\lim\limits_{x\to 3/2^{+}} \operatorname{int}(2x-1)$

g) $\lim\limits_{x\to\infty} \dfrac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}}$

h) $\lim\limits_{x\to\infty} \dfrac{4n^3}{n^2+10000n} =$

i) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{4x}$

j) $\lim\limits_{x\to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

k) $\lim\limits_{x\to\infty} \dfrac{x\sin x+2\sin x}{2x^2}$

l) $\lim\limits_{x\to -2} \dfrac{x^2+1}{3x^2-2x+5}$