Your Turn 1: Suppose you flip a coin three times. Find the probability that you get the following number of heads. A tree diagram (below right) is useful for listing all of the possible outcomes.

		1 <sup>et</sup> Flip	2 <sup>nd</sup> Flip	3 <sup>rd</sup> Flip	Result
# of heads	Probability	~	h<	$h_{t}$	<ul> <li>{h,h,h}</li> <li>{h,h,t}</li> </ul>
0	1/8 = 0.125	n	_t <	_h	{h,t,h} {h,t,t}
1	3/8 = 0.375 3/8 = 0.375		_h <	h	{t,h,h}
3	1/8 = 0.125	t<		h	{t,n,t} {t,t,h}
a de la companya de la	10		1-	t	• {t,t,t}

 Your Turn 2: You enter a lottery by purchasing 1 of 200 tickets. The organizers randomly select one ticket for the grand prize and then five other tickets for small prizes.

- What is the probability that you win the grand prize?  $\frac{1}{200} = 0.005$
- What is the probability that you win a prize of some type?  $\frac{6}{200} = 0.03$
- What is the probability that you don't win anything?  $\frac{194}{200} = 0.97$
- Relative Frequency Approximation of Probability: Conduct or observe a procedure and then count the number of times that event A occurs. Based on these results, P(A) is approximated by

 $P(A) \approx \frac{\text{number of times } A \text{ occurred}}{\text{number of times the trial was repeated}}$ 

 Example: Assume Shaquille O'Neal made 5750 of his last 10895 free-throw attempts. Based on this data, estimate the probability that he makes his next free-throw.

Answer:  $P(\text{he makes it}) \approx \frac{5750}{10895} = 0.528$ 

· Your Turn: Of the 200 people that walked by your store on Church Street over the past hour, 15 of them entered your store. Estimate the probability that the next person coming down the street will walk into your store.

 $P(\text{next person enters}) \approx \frac{15}{200} = 0.075$ 

s**e**n a a a terring spins has 



2. Your Turn: If you pick one card from a deck, what is the probability that it is a Heart given that it is a Jack?

There are 4 Jacks and only one is a Heart. So the probability is  $\frac{1}{4} = 0.25$ .

3. If you pick two cards from a deck without replacement, what is the probability that the second card is a Jack given that the first card you picked was a Jack?

Answer: Because you are told the first card was a Jack, this is a conditional probability. You want to find  $P(\text{Jack}_2 | \text{Jack}_1)$ .

Since you are holding a Jack, there are only 3 of them left in a deck that now contains only 51 cards. So  $P(\text{Jack}_2 | \text{Jack}_1) = \frac{3}{51} \approx 0.0588$ 

- 4. Your Turn: If you pick two cards from a deck without replacement, what is the probability that the second card is a Jack given that the first card you picked was a Queen? If you are holding a Queen that means there still 4 Jacks in the remaining 51 cards. The probability is <sup>4</sup>/<sub>51</sub> ≈ 0.0784.
- 5. Your Turn: If you pick two cards from a deck with replacement (meaning the first card is returned to the deck before you pick your second card), what is the probability that the second card is a Jack given that the first card is a Jack?

Since the first Jack went back into the deck after being picked, it does not affect your probability of getting another Jack. That means there are still 4 Jacks in the 52 cards. The probability is  $\frac{4}{52} \approx 0.0769$ .

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- 2. Your Turn: Find the probability that the device tests negative on a person with cancer. This is called a *False Negative*. P(Negative Test | With Cancer) =  $\frac{2}{200} = 0.01$  or 1%.
- 3. What do your answers to parts (a) and (b) suggest about this particular screening device. It is probably calibrated in such a way to make fewer False Negatives than False Positives. It is difficult to reduce the probability of one without increasing the probability of the other (good talking point later with Type I and Type II errors)

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 If one card is drawn from a standard deck of cards find the probability that it is a King or a Heart. Your Turn: Since these events are not mutually exclusive (you can draw a King and a Heart).

Since these events are not mutually exclusive (you can draw a King and a Heart),  $P(K \text{ or } H) = P(K) + P(H) - P(K \text{ and } H) = \frac{4}{82} + \frac{13}{52} - \frac{1}{52} = \frac{46}{52} \approx 0.308$ 

 The following table gives the gender and class standing for 2,400 student from a small private New England college. The totals are helpful in answering these questions.

		Class Standing					
	Freshman	Sophomore	Junior	Senior	Graduate	Totals	
Male	255	260	200	165	160	1,040	
Female	320	265	275	260	240	1,360	
Totals	575	525	475	425	400	2,400	

If one student is randomly selected, what is the probability of selecting

(a) a male or a Freshman?

Answer: Since these events are not mutually exclusive (you can get a male and a Freshman),

$$P(M \text{ or } Fr) = P(M) + P(Fr) - P(M \text{ and } Fr) = \frac{1040}{2400} + \frac{575}{2400} - \frac{255}{2400} = \frac{1360}{2400} \approx 0.567$$

(b) a female or a Senior?

Your Turn:

Since these events are not mutually exclusive (you can get a female and a Senior),  $P(F \text{ or } Sr) = P(F) + P(Sr) - P(F \text{ and } Sr) = \frac{1360}{2400} + \frac{425}{2400} - \frac{260}{2400} = \frac{1525}{2400} \approx 0.635$ 

- (c) a Junior or a Senior? Your Turn:
  - Tour Turn:

Since these events are mutually exclusive (you can't get a Junior and a Senior),  $P(Jr \text{ or } Sr) = P(Jr) + P(Sr) = \frac{475}{2400} + \frac{425}{2400} = \frac{900}{2400} = 0.375$ 

### Page 76

 If you randomly select two marbles without replacement what is the probability that you get a red marble on the first and a green marble on the second selection.

#### Your Turn:

Since the marbles are selected without replacement, the events are **dependent** and  $P(R_1 \text{ and } G_2) = P(R_1) \cdot P(G_2|R_1) = \frac{4}{8} \cdot \frac{4}{7} = \frac{2}{7} \approx 0.286$ 

Suppose you have an alarm clock that works 90% of the days it is set. What is the probability of it working 5 days in a row?

Your turn:  $(0.9)^5 \approx 0.590$ 

#### Page 77

If you select 4 marbles without replacement, what is the probability of getting all red marbles.
 Your Turn:

 $P(R_1 \text{ and } R_2 \text{ and } R_3 \text{ and } R_4) = \frac{4}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \approx 0.0143$ 

# Page 78

- Your Turn: Assume that 20% of all college students know how to perform a correlation test. There is
  a randomly selected group of 5 students working on a project which requires a correlation test.
  - (a) What is the probability that at least one student knows how to run a correlation test? Let A = at least one has the knowledge. Then A = none have the knowledge. P(A) = (.8)<sup>5</sup>, so P(A) = 1 − (.8)<sup>5</sup> = 0.67232 ≈ 0.672.
  - (b) In the next chapter we will learn how to calculate the probability that exactly one of these students knows how to run a correlation test. That probability is 0.410.
  - (c) What is the probability that all 5 students know how to run a correlation test? You need all 5 to have this knowledge, so the probability is (.20)<sup>5</sup> = 0.00032
  - (d) Which probability, part (a), (b), or (c), is most relevant to the task of completing the group project? Since this is a group project, it is only important that one or more students know how to run a correlation test. As such, the probability from part (a) is the relevant one.

# Page 95

Your Turn, Carnival Game: You play a \$5 carnival game where you fish for prizes. There are 100 plastic fish circulating in the pond. Each fish contains a small piece of paper with a prize code printed on it. There are 10 red fish with a \$10 prize, 20 blue fish with a \$5 prize, 30 green fish with a \$2 prize, and 40 orange fish with a \$1 prize. You pay the \$5, drop your line, and hope for the best. What is the expected value of this game to you?

Outcomes	value = $x$	P(x)	$x \cdot P(x)$
red	5	10/100 = .1	0.50
blue	0	20/100 = .2	0.00
green	-3	30/100 = .3	-0.90
orange	-4	40/100 = .4	-1.60
			E = -2.00

You take your nephew to the carnival and he wants to play this game. Instead of paying the \$5 for him to play, you offer him cash instead. What is a fair cash settlement? \$3 would be fair, but anything greater than \$3 and less than \$5 would be good for both of you. It depends on what you call fair.

#### Page 96

Your Turn: A huge can of mixed nuts contain 60% cashews and you randomly select 20 nuts. Let x denote the number of cashews you get. Verify that x follows a binomial probability distribution and determine n, p, and q.

(a) There are a fixed number of trials, n=20. (b) The outcomes aren't officially independent; if we are certain there are 20% cashews then drawing one cashew does change the probability of drawing another. However, this is a huge can of nuts and you're only picking 20 so we'll assume this is less than 5% of the total number of nuts (a point worth discussing) and consider the trials independent. (c) There are only two outcomes: cashew and not cashew. (d) Again assuming the events are independent, the probability of success remains the same for all trials.

### Page 97

44 N.C. N.C. N.C. N.C.

2. Your Turn: What is the probability that you get exactly 4 questions correct?  $P(x = 4 \mid n = 15, p = 0.2) = \frac{15!}{111.4!} \cdot (.2)^4 \cdot (.8)^{11} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1} \cdot (.2)^4 \cdot (.8)^{11}$ 

 $= 1365 \cdot (.2)^4 \cdot (.8)^{41} = 0.1876041715 \rightarrow 0.188$ 

Binomial Probability Tables (Table 1, pages 337 to 339):

If you want the probability of x successes in n trials with a probability of a single success = p, then

- n determines which table to use.
- p determines the appropriate column.
- x determines the row.
- The value given in the table is the probability you seek = P(x|n, p).

Examples - Using the Binomial Probability Tables: You are taking a 15-question multiple choice quiz and each question has 5 options (a,b,c,d,e) and you randomly guess at every question. Use the binomial tables to answer the following questions.

- What is the probability that you get exactly two questions correct? P(x = 2 | n = 15, p = 0.2) = 0.231 (same as above but much easier)
- Your Turn: What is the probability that you get exactly four questions correct? P(x = 4 | n = 15, p = 0.2) = 0.188
- What is the probability that you get six or more questions correct?
   P(x ≥ 6 | n = 15, p = 0.2) = 0.043 + 0.014 + 0.003 + 0.001 + \* + ... ≈ 0.061.
- 4. Your Turn: What is the probability that you get less than 4 correct?  $P(x < 4 \mid n = 15, p = 0.2) = 0.250 + 0.231 + 0.132 + 0.035 = 0.648$

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# Chapter 4: Summary Worksheet - Solutions

1. Use the following data for the 100 Senators from the 112<sup>th</sup> Congress of the United States.

	Republican	Democrat	Independent
Male	42	39	2
Female	5	12	0

If one Senator is randomly selected, find the probability of

(a) getting a non-Republican:

By the law of compliments: Since there are a total of 47 Republicans, the probability of getting a Republican is  $\frac{47}{100} = 0.47$ . Therefore, the probability of getting a non-Republican is 1 - 0.47 = 0.53. You could also count up all the non-Republicans and get 53 of them.

- (b) getting a male or a Republican.
  By the formula: P(Male or Republican) =
  P(Male) + P(Republican) P(Male and Republican) = 0.83 + 0.47 0.42 = 0.88.
  By counting: There are 83 males and 5 Republicans that are not males, so there are 88 in the category of male or Republican and the probability is 0.88.
- (c) getting a Democrat given that this Senator is a female. Ie. Find P(Democrat|Female). Given that the senator is a female, there are only 17 total and 12 of these are democrats so  $P(\text{Democrat}|\text{Female}) = \frac{12}{17} \approx 0.706$ .
- (d) getting a female given that this Senator is a Democrat. Ie. Find P(Female|Democrat). Given that the Senator is a Democrat, there are only 51 total and 12 of these are female so  $P(\text{Female}|\text{Democrat}) = \frac{12}{51} \approx 0.235$ .

# 2. A box of 8 marbles has 5 red marbles, 2 green marbles, and 1 blue marble.

- (a) Find the probability of selecting 2 red marbles if the first selection is replaced before the next selection is made. Round your answer to 3 significant digits Since there is replacement, these events are independent and  $P(\text{red and red}) = P(\text{red on first}) \cdot P(\text{red on second}) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64} \approx 0.391.$
- (b) Find the probability of selecting 2 red marbles if the the first selection is not replaced before the next selection is made. Round your answer to 3 significant digits Since there is no replacement, these events are dependent and  $P(\text{red and red}) = P(\text{red on first}) \cdot P(\text{red on second}|\text{red on first}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} \approx 0.357.$
- (c) Find the probability of selecting a red marble followed by a green marble if the first selection is not replaced before the next selection is made. Round your answer to 3 significant digits Since there is no replacement, these events are dependent and  $P(\text{red and green}) = P(\text{red on first}) \cdot P(\text{green on second} \mid \text{red on first}) = \frac{5}{8} \cdot \frac{2}{7} = \frac{10}{56} \approx 0.179.$

- 3. Scott shaves his face every morning. For the last 200 days, he has cut himself 36 times.
  - (a) Find the relative frequency approximation to the probability that Scott will cut himself when he shaves tomorrow.

The relative frequency approximation is  $\frac{36}{200} = 0.18$ .

- (b) Use this approximation to determine the probability that he cuts himself two days in a row. Assume the events are independent.
  While one might argue that he is less likely to cut himself on day 2 after cutting himself on day 1, we are to assume the events are independent. And, P(cut on first day and cut on second day) = 0.18 · 0.18 = 0.0324
- (c) Last year, he went the entire month of August (31 days) without cutting himself. Just last week, he cut himself 3 days in a row. Which event is more unusual? I.e., which event has a lower probability of occurring assuming the probability from part (a) is accurate?

P( cut-free in August  $) = 0.82^{31} \approx 0.00213$ P( three consecutive cuts  $) = 0.18^3 \approx 0.00583$ Going the entire month of August cut-free is more unusual.

4. A preliminary cancer screening device is tested for its ability to accurately determine whether a subject actually has cancer or not. The results of this test (total 400 trials) are summarized in the table below. Note: A positive test result indicates that cancer **is** present.

	Did the person actually have cancer?		
	Yes	No	
positive test result	198	20	
negative test result	2	180	

Answer the following questions based on the data found in the table.

(a) Find the probability that a cancer-free person tests positive? (False Positive)

 $P(\text{Tests Positive} \mid \text{No Cancer}) = \frac{20}{200} = 0.10$ 

(b) If a cancer-free person has 10 cancer screenings in 10 years, what is the probability that this person will experience **at least one** false positive. Here, A is the event that this cancer-free person has at least one positive. Then  $\bar{A}$  is no false positives or 10 negatives.  $P(\bar{A}) = (0.9)^{10} \approx 0.349$  and  $P(A) = 1 - P(\bar{A}) = 1 - 0.349 = 0.651 \dots$  That's pretty high. 12a Pirates were huge in the early 1900's while mutants were barely mentioned. Around 1945 (radiation days) mutant popularity sky-rocketed passing pirates around 1970. Around 1980 mutants started to diminish and pirates started to recover. Around 2003 (release of *Pirates of the Caribbean* and headline news about real pirates off the coast of Africa) pirates resumed their popularity over mutants.

13 In the years 2007 to the beginning of 2010, there was a trend where concerns for the environment decreased as concerns for energy increased. This could be due to the increasing cost of oil, an economic downturn, and uneasy relationships with oil-producing nations. In April of 2010, the Deepwater Horizon oil spill occurred. This spill is the largest offshore spill in U.S. history and clearly changed peoples opinions as to the priorities regarding the environment and energy. However, that change was short-lived and by March of 2011 the opinions were back to where they were before the crisis. In 2012, there is a pretty even split of opinion. **Trick:** The differences are exaggerated by starting the *y*-axis at 30% instead of zero.

# Chapter 4

**1a** The sample space is {bb, bg, gb, gg}.

1b There are 4 possible outcomes and two of these have one boy and one girl. So the probability of having one boy and one girl is  $\frac{2}{4} = 0.5$ .

1c There are 4 possible outcomes and three of these will have at least one girl. So the probability of having at least one girl is  $\frac{3}{4} = 0.75$ .

1d There are 4 possible outcomes and only one has no girls in it. So the probability is  $\frac{1}{4} = 0.25$ .

**3a** There are 1,000 tickets and only 1 is the grand prize ticket. So your probability is 1/1000 = 0.001.

**3b** There are 1,000 tickets and 8 of these will produce some type of prize. So your probability is 8/1000 = 0.008

**3c** It is tricky because any one ticket could fall into more than one category. For example, your ticket could be a grand prize winner **and** a small prize winner.

**5a** 923/1567 = 0.589.

**5b** Based on the historical data and using the relative frequency approach to estimating a probability, the estimated probability would again be 0.589.

7 P(next one sold fails)  $\approx \frac{24}{4000} = 0.006$ 

**9a** The prediction was correct 102 + 205 = 307 times and it was wrong 18+40 = 58 times.

**9b** There are 365 days and the prediction was correct 307 times so the probability is 307/365 = 0.841.

**9c** Using the historical data and the relative frequency approach to estimating probabilities, the estimated probability is 307/365 = 0.841.

10a This not a conditional probability. The prediction was correct 307 times out of 365. So the probability is  $\frac{307}{365} \approx 0.841$ .

10b Since we are given that it was predicted to rain, there are only 120 options in our sample space. Of these, it rained in 102 of these cases. So the probability is  $\frac{102}{120} = 0.850$ .

10c Since we are given that it was predicted not to rain, there are only 245 options in our sample space. Of these, it did not rain 205 of these cases. So the probability is  $\frac{205}{245} \approx 0.837$ .

10d Comparing the two previous answers, the forecast is better at predicting rain because there is an 85.0% success rate at this but only an 83.7% success rate at predicting no rain.

12a If you know that he has no aces then there are 4 aces left in the deck which now only has 44 cards in it. So, the probability that you get an ace is 4/44 = .0909.

**13a** Since you are already holding four cards, this is a conditional probability. There are 48 cards left but only 9 hearts (because you have 4 of them). Therefore  $P(\heartsuit) = 9/48 = 0.1875$ .

13b Since you are already holding four cards, this is a conditional probability. There are 48 cards left and you would be happy with a 2 or a 7 of any suit. Since there are four 2's and four 7's left in the deck, there are eight ways to get what you want and  $P(\text{straight}) = 8/48 \approx 0.167$ .

14a Mutually exclusive, you can not roll a 6 and a 2 on a single roll.

14d Not mutually exclusive. It is possible (and likely) that a vegetarian meal will contain vegetables.

**15a** There are two ways you can do this. You can get a 1 on the red die and a 2 on the white **or** a 2 on the red die and a 1 on the white. These events are mutually exclusive. So,  $P(R1 \text{ and } W2 \text{ or } R2 \text{ and } W1) = \frac{1}{36} + \frac{1}{36} = \frac{2}{36} \approx 0.0556$ .

**15b** You could use the addition rule and count all the different ways to roll a total not equal to 3. However, it is a lot easier to use the compliments rule. If A = a total that is not 3, then  $\bar{A} = a$  total of 3. You found  $P(\bar{A})$  in the last problem. So,  $P(A) = 1 - P(\bar{A}) = 1 - \frac{2}{36} = \frac{34}{36} \approx 0.944$ .

16 Here is the same table with the totals calculated:

	Has a ski pass at					
	Bolton	Stowe	Smuggler's	Sugarbush	No Pass	Totals
Has a Car	18	12	10	30	25	95
Does not have a Car	24	5	4	12	60	105
Totals	42	17	14	42	85	200

16a There are 42 students with a pass at Bolton, 17 at Stowe, 14 at Smuggler's, 42 at Sugarbush, and 85 with no pass. Totalling these numbers you get 200. Since there are only 200 students all together you can conclude that none of them have more than one pass otherwise your total would have been greater than 200.

16b Since these are mutually exclusive events (from the previous problem),  $P(\text{Stowe or Sugarbush}) = P(\text{Stowe}) + P(\text{Sugarbush}) = \frac{17}{200} + \frac{42}{200} = \frac{59}{200} = 0.295.$ 

16c Since these are not mutually exclusive, P(car or Sugarbush) = P(car) + P(Sugarbush) - P(car and Sugarbush) = 95/200 + 42/200 - 30/200 = 107/200 = 0.535.

**16d** If A = car or ski pass, then  $\bar{A} = \text{no car and no pass}$ .  $P(\bar{A}) = 60/200$ , so P(A) = 1 - 60/200 = 140/200 = 0.700.

**18a** Since the drawing is done without replacement, the probability of getting a Jack on the second draw is affected by whether or not you got a Jack on the first draw. Thus, the events are **dependent**.

18b If you have no idea how much of the store's milk supply is spoiled, then finding the first one is spoiled increases the probability that the second one is spoiled. If you somehow know exactly how many gallons are spoiled and how many are not, then selecting a spoiled one might decrease the probability that the second one is spoiled (there is one fewer spoiled gallon remaining when you pick the second one). Either way, the events are **dependent**.

**18c** Recognizing that this is a fair coin, getting *heads* on the first toss does not change the probability of getting *heads* on the second toss. The events are **independent**.

**19a** We use the multiplication rule for independent events:  $P(Jack then Jack) = P(Jack) \cdot P(Jack) = 4/52 * 4/52 = 0.00592.$ 

**19b** We use the multiplication rule for independent events:  $P(Jack then Queen) = P(Jack) \cdot P(Queen) = 4/52 \cdot 4/52 = 0.00592.$ 

**19c** We use the multiplication rule for independent events:  $P(Jack then Heart) = P(Jack) \cdot P(Heart) = 4/52 * 13/52 = 1/52 = 0.0192.$ 

**21b** The day of the first person does not matter. The probability that the second person has the same day is 1/7 and the probability that the third person has the same day is 1/7 and the same goes for the fourth

person. So, the probability that all 4 have were born on the same day of the week is  $\left(\frac{1}{7}\right)^3 = 0.00292$ .

**22a** There are two ways for this to happen. The first could be red and the second yellow **or** the first is yellow and the second is red. Since these are mutually exclusive events, we add the probabilities. P({red and yellow} **or** { yellow and red}) =  $\frac{3}{12}\frac{5}{12} + \frac{5}{12}\frac{3}{12} = 0.208$ .

**22c** This is a natural extension of the multiplication rule with dependent events. P(yellow and yellow and yellow and yellow) =  $\frac{5}{12} \frac{4}{11} \frac{3}{10} \frac{2}{9} \frac{1}{8} = 0.00126$ .

**23a** Since we are told these are two different senators, the sampling is done without replacement and the events are dependent. So, P(male republican **and** male republican) =  $\frac{42}{100}\frac{41}{99} = 0.174$ 

**23b** Since we are told these are two different senators, the sampling is done without replacement and the events are dependent. So, P(democrat **and** republican) =  $\frac{51}{100}\frac{47}{99} = 0.242$ 

**24a** The probability of getting a spark plug that is not defective is 1 - 0.02 = 0.98.

**24b** The probability that all 4 spark plugs are not defective is  $(0.98)^4 = 0.922$ .

**24c** The complement of *at least one defective* is none defective or all are defect-free. The probability that all are defect free was found in part b as 0.922. So the probability that at least one is defective is 1-.922 = 0.078.

26 The complement of the event *at least one* is none. In this case, none means that that the lie detector accurately detects all 10 lies. The probability of this is  $(0.95)^{10} = 0.599$ . But this is the probability that all 10 lies are detected. If this doesn't happen then at least one lie went undetected. So the probability of at least one lie going undetected is 1 - 0.599 = 0.401.

**28a** You need all three to have jumper-cables so the probability is  $(.25)^3 = 0.015625 \approx 0.0156$ 

**28b** Let A = at least one has jumper cables. Then  $\overline{A}$  = none have jumper cables.  $P(\overline{A}) = (.75)^3$ , so  $P(A) = 1 - (.75)^3 = 0.578125 \approx 0.578$ .

**28c** You don't need all three to have jumper-cables. You only need one or, more precisely, at least one of them, to have jumper-cables. So the second probability is more relevant.

# Chapter 5

1a The probabilities are all between 0 and 1, and they sum to 1, so this is a probability distribution.

**1b** The mean value is  $\mu = \sum (x \cdot P(x))$  (see table below) and the mean is **2**.

x = # of heads	P(x)	$x \cdot P(x)$
0	1/16	0
1	4/16	4/16
2	6/16	12/16
3	4/16	12/16
4	1/16	4/16
sum $(\Sigma)$		32/16

1c From the last problem we see that the expected value is just the mean of the probability distribution = 2 heads. You probably could have guessed this without all the math.

3a Each probability is between 0 and 1, and the probabilities add to 1, so this is a probability distribution.

**3b** From the table below, you can see that the expected value is -1450/200 = -7.25. So the expected value of this raffle to me is **-\$7.25**.

Outcomes	value $= x$	P(x)	$x \cdot P(x)$
Win Grand Prize	190	1/200	190/200
Win a Second Prize	90	2/200	180/200
Win a Third Prize	40	3/200	120/200
Win Nothing	-10	194/200	-1940/200
Sum $(\Sigma)$			E = -1450/200

**5** If you had bought the warranty, your cost would have been \$160. If you don't buy it, you have to determine your expected cost.