~~(9a) $f(x) = x$~~

HW

(1) $f(x) = x^2 - 6x + 1 \quad c = 0$

$f'(x) = 2x - 6$

$f'(0) = -6$

(2) $f(x) = x^{-1} - 3x^{-2} + 4x^{-3} \quad c = -1$

$-x^{-2} + 6x^{-3} - 12x^{-4}$

$f'(x) = -\frac{1}{x^2} + \frac{6}{x^3} - \frac{12}{x^4}$

$f'(-1) = -\frac{1}{1} - \frac{6}{1} - \frac{12}{1} = -19$

(3) $f(x) = 3x^{\frac{1}{2}} - \frac{1}{\sqrt[3]{x}} \quad c = 1$

$3x^{\frac{1}{2}} - x^{-\frac{1}{3}}$

$\frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{4}{3}}$

$\frac{3}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^4}} \quad c = 1$

$\frac{3}{2\sqrt{1}} + \frac{1}{3\sqrt[3]{1^4}} = \frac{3}{2} + \frac{1}{3}$

$\frac{9}{6} + \frac{2}{6}$

$\frac{11}{6}$

(4) $y' = 0$ constant

(5) $y = \frac{1}{6}x^2 + \frac{2}{6}$

$\boxed{y' = \frac{1}{3}x}$ you could have use the quotient rule, but why?

(6) $y = \frac{1}{a}\left(x^2 - \frac{1}{b^2}x + c\right)$

#'s or Constants

$\frac{1}{a}\left(2x - \frac{1}{b^2} + 0\right)$

$y' = \frac{2}{a}x - \frac{1}{ab^2}$

(7) $\frac{3x^2(0) - 8(6x)}{9x^4}$ $\quad \frac{-48x}{9x^4}$

$y' = \boxed{\frac{-16}{3x^3}}$

(8) $\frac{-9}{27x^6} = \frac{-1}{3x^6}$

$y' = \frac{3x^6(0) - (-(18x^5))}{9x^{12}}$ $\quad \frac{18x^5}{9x^{12}} = \frac{2}{x^7}$

(10) $y = \frac{4}{3}x^2 - \frac{5}{3}x + 2$

$$\boxed{y' = \frac{8}{3}x - \frac{5}{3}}$$

(11) $y = \frac{x^3}{2x} - \frac{6x}{2x} + \frac{2}{2x}$

$\frac{1}{2}x^2 - 3 + \frac{1}{x}$

$\frac{1}{2}x^2 - 3 + x^{-1}$

$y' = x + 0 - x^{-2} \Rightarrow x - \frac{1}{x^2}$

(12) $y =$ we could this

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$\frac{\cancel{(x+2)}(x^2 - 2x + 4)}{\cancel{(x+2)}}$$ Hole in graph

$y = x^2 - 2x + 4$

$y' = 2x - 2$ $\quad x \neq -2$

(12) Alternative

$$y = \frac{x^3+8}{x+2} \qquad \frac{(x+2)(3x^2) - (x^3+8)(1)}{(x+2)^2}$$

$$\frac{3x^3+6x^2-x^3-8}{(x+2)^2} = \frac{2x^3+6x^2+8}{(x+2)^2}$$

This simplifies to the other. (Synthetic division Xander)

(13) $y = x^4 - \frac{3}{2}x^3 + 5x^2 - 6x - 2$

$$\boxed{y' = 4x^3 - \frac{9}{2}x^2 + 10x - 6}$$

(14) $\frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{10x}{x^2} - \frac{5}{x^2}$

$$x - 3 + \frac{10}{x} - \frac{5}{x^2}$$

$$x - 3 + 10x^{-1} - 5x^{-2}$$

$$1 + 0 - 10x^{-2} + 10x^{-3}$$

$$\boxed{1 - \frac{10}{x^2} + \frac{10}{x^3}}$$

15 Product rule

$(x^2+4x)(2x-1)$

$\boxed{y' = (2x-1)(2x+4) + (x^2+4x)(2)}$

$\downarrow$

This simplifies to

$4x^2+6x-4 + 2x^2+8x$

$\Rightarrow 6x^2+14x-4$

So if we expand the original it's

$(x^2+4x)(2x-1)$ $2x^3+8x^2-x^2-4x$

$2x^3+7x^2-4x$

$y' = 6x^2+14x-4$

Which is easier?

16 $(x-2)^3$ Want to learn the chain rule? $\frac{d}{dx} f(g(x)) = f'(g(x))\,g'(x)$

$\boxed{3(x-2)^2(1)}$

Don't worry about this one

(17) $y = x^{\frac{1}{3}} - x^{\frac{2}{3}}$

$y' = \frac{1}{3}x^{-\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{3}}$

$$\boxed{y' = \frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x}}}$$

(18) $y = \frac{x^4}{x} - \frac{2x^3}{x} - \frac{5x^2}{x} - \frac{4x}{x} + \frac{4}{x}$

$x^3 - 2x^2 - 5x - 4 + 4x^{-1}$

$3x^2 - 4x - 5 + 0 - 4x^{-2}$

$y' = 3x^2 - 4x - 5 - \frac{4}{x^2}$

We could get these all over

$x^2 \rightarrow \frac{3x^4 - 4x^3 - 5x^2 - 4}{x^2}$

if you chose to use the quotient rule on $\frac{x^4 - 2x^3 + 5x^2 - 4x + 4}{x}$

$$\frac{x(4x^3 - 6x^2 + 10x - 4) - (x^4 - 2x^3 + 5x^2 - 4x + 4)(1)}{x^2}$$

$$\frac{4x^4 - 6x^3 + 10x^2 - 4x - x^4 + 2x^3 - 5x^2 + 4x - 4}{x^4}$$

$y' = \frac{3x^4 - 4x^3 + 5x^2 - 4}{x^2}$ ✓

(19) $y = (x^2-4x-6)(x^3-5x^2-3x)$

Product rule

$(x^3-5x^2-3x)(2x-4)+(x^2-4x-6)(3x^2-10x-3)$

OK... $2x^4-10x^3-6x^2$
$-4x^3+20x^2+12x$

$2x^4-14x^3+14x^2+12x$

$3x^4-12x^3-18x^2$
$-10x^3+40x^2+60x$
$-3x^2+12x+18$

$2x^4-14x^3+14x^2+12x+3x^4-22x^3+19x^2+72x+18$

$y' = \boxed{5x^4-36x^3+33x^2+84x+18}$

I'm not ck'ing.

(20) $y = \dfrac{3x-2}{2x+3}$ $\quad y' = \dfrac{(2x+3)(3)-(3x-2)(2)}{(2x+3)^2}$

$\dfrac{6x+9-6x+4}{(2x+3)^2} = \boxed{\dfrac{13}{(2x+3)^2} = y'}$

21 $$\boxed{\frac{(x^2-1)(2x-4)-(x^2-4x-2)(2x)}{(x^2-1)^2}}$$

$2x^3-2x$

$-4x^2+4$

$2x^3-4x^2-2x+4 \quad -(2x^3-8x^2-4x)$

$2x^3-4x^2-2x+4-2x^3+8x^2+4x$

$$\boxed{4x^2+2x+4}$$

22 $y=\frac{x-1}{x^{\frac{1}{2}}}$ $\qquad \frac{x^{\frac{1}{2}}(1)-(x-1)(\frac{1}{2}x^{-\frac{1}{2}})}{(x^{\frac{1}{2}})^2}$

$x^{\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}}{x}$ $\qquad \boxed{\frac{\frac{1}{2}\sqrt{x}+\frac{1}{2\sqrt{x}}}{x}}$

to go further

$\left(\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}\right)\frac{1}{x}$

$\left(\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}\right)x^{-1}$

$\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ $\qquad \boxed{\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{x^3}}}$

(23) $\dfrac{x^2-x+1}{x^{1/3}}$

$$y' = \boxed{\dfrac{x^{1/3}(2x-1) - (x^2-x+1)\left(\frac{1}{3}x^{-2/3}\right)}{x^{2/3}}}$$

I'm not Simplifying

(24) Product rule with a quotient rule

$$y = \left(\frac{x-3}{x-4}\right)(3x+2)$$

$$(3x+2)\left(\frac{(x-4)\cdot 1 - (x-3)(1)}{(x-4)^2}\right) + \left(\frac{x-3}{x-4}\right)3$$

$$\frac{(3x+2)(x-4-x+3)}{(x-4)^2} + \frac{3x-9}{x-4}$$

$$y' = \boxed{\frac{(3x+2)(-1)}{(x-4)^2} + \frac{3x-9}{x-4}}$$

25 $y = \frac{1}{(x^2-5)^2} \rightarrow \frac{1}{(x^2-5)(x^2-5)}$

$\frac{1}{x^4-10x^2+25}$

$\frac{(x^4-10x^2+25)0 + 1(4x^3-20x)}{(x^2-5)^2}$

$\frac{4x^3-20x}{(x^2-5)^2} \rightarrow \frac{4x(x^2-5)}{(x^2-5)^2}$

$\boxed{y' = \frac{4x}{x^2-5}}$

---

26 $y = \frac{x+k}{x-k}$ $\frac{(x-k)1 - (x+k)1}{(x-k)^2}$

$\frac{x-k-x-k}{(x-k)^2} = \boxed{\frac{-2k}{(x-k)^2}}$

27

$$y = \frac{x^2 - k^2}{x^2 + k^2}$$

$$\frac{(x^2+k^2)(2x) - (x^2-k^2)(2x)}{(x^2+k^2)^2}$$

$$\frac{2x^3 + 2k^2x - 2x^3 + 2k^2x}{(x^2+k^2)^2}$$

$$y' = \frac{4k^2x}{(x^2+k^2)^2}$$

28 $f(x) = \frac{x^3}{x-1}$ @ $(2, 4)$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{x-1}$$

$$f'(2) = \frac{(1)(4) - (4)}{2-1} = \frac{0}{1} = 0$$

$y = 4$

29 $f(x) = (x-2)(x^2-3x-1)$ @ $(-1,-9)$

$f'(x) = \cancel{x-2}\ (x^2-3x-1)(1) + (x-2)(2x-3)$

$\cancel{f'(-1) = (-3)(1+3-1)}$

$f'(-1) = (1+3-1)(1) + (-3)(-2-3)$

$3 + 15 = 18$

$\boxed{y+9 = 18(x+1)}$

30 $\dfrac{x^2-4x+2}{2x-1}$ @ $(2, -\frac{2}{3})$

$f'(x) = \dfrac{(2x-1)(2x-4) - (x^2-4x+2)(2)}{(2x-1)^2}$

$f'(2) = \dfrac{(3)(0) - (4-8+2)(2)}{(2(2)-1)^2}$

$y'(2) = \dfrac{+4}{9}$

$\boxed{y+\frac{2}{3} = \frac{4}{9}(x-2)}$

(31) $f(x) = \dfrac{-1}{(2x-3)^2}$ @ $\left(0, -\frac{1}{9}\right)$

~~$f'(x)$~~ $=$

$f(x) = \dfrac{-1}{4x^2-12x+9}$

$f'(x) = \dfrac{\text{bottom}(0) - (-1)(8x-12)}{(2x-3)^4}$

$f'(x) = \dfrac{8x-12}{(2x-3)^4}$ $\quad f'(0) = \dfrac{-12}{81}$

$f'(0) = \dfrac{-4}{27}$

$$\boxed{y + \frac{1}{9} = -\frac{4}{27}(x-0)}$$

(32) $f(x) = 4x^2 - 8x - 3$ @ $\left(\frac{1}{2}, -6\right)$

$f'(x) = 8x - 8$

$f'\left(\frac{1}{2}\right) = 4 - 8 \rightarrow -4$

Slope of normal line $\frac{1}{4}$

$$\boxed{y + 6 = \frac{1}{4}\left(x - \frac{1}{2}\right)}$$

(33) $f(x) = \dfrac{x^{\frac{1}{2}}+4}{x^{\frac{1}{2}}}$ we can write as $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \dfrac{4}{x^{\frac{1}{2}}}$ (4,3)

$f(x) = 1 + 4x^{-\frac{1}{2}} \longrightarrow f'(x) = -2x^{-\frac{3}{2}}$

$f'(4) = \dfrac{-2}{\sqrt{4}^3}$ $\quad \dfrac{-2}{\cancel{64}\,8} \rightarrow -\dfrac{1}{32}$ ~~slope~~ slope of normal 32

$y - 3 = 32(x-4)$

(34) Slope $= 0$

$$y = \frac{x^2}{x^2-4} \qquad y' = \frac{(x^2-4)(2x)-(x^2)(2x)}{(x^2-4)^2}$$

$$\frac{0}{\#} = 0 \qquad \frac{2x^3-8x-2x^3}{(x^2-4)^2} \Rightarrow \frac{-8x}{(x^2-4)^2}$$

$$-8x = 0 \qquad x = 0$$

$$y(0) = \frac{0^3}{0^2-4} \rightarrow 0 \qquad (0,0)$$

We knew this.
Why? How?
what are the x-intercepts of f(x)
does it have amultiplicity > 1 ?

(35) $$f(x) = \frac{4x}{x^2+4} \rightarrow f'(x) = \frac{(x^2+4)4 - 4x(2x)}{(x^2+4)^2}$$

$$f'(x) = \frac{4x^2+16-8x^2}{(x^2+4)^2} = \frac{16-4x^2}{(x^2+4)^2}$$

$$16-4x^2 = 0$$
$$16 = 4x^2$$
$$4 = x^2 \qquad x = \pm 2$$

$$f(-2) = \frac{-8}{8} = -1 \qquad (-2,-1)$$

$$f(2) = \frac{8}{8} = 1 \qquad (1, 2)$$

36

$f(x) = x^3 - 9x$

$f'(x) = 3x^2 - 9$

$f'(x) = 0$

$3x^2 - 9 = 0$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$f(-\sqrt{3}) = (-\sqrt{3})^3 - 9(-\sqrt{3})$

$(-3^{\frac{1}{2}})^3 + 9\sqrt{3}$

$(-1 \cdot 3^{\frac{1}{2}})^3 = -1 \cdot 3^{\frac{3}{2}}$

$= -1\sqrt{3^3}$

$\sqrt{9} \cdot \sqrt{3}$

$-3\sqrt{3} + 9\sqrt{3}$

$6\sqrt{3}$

$\boxed{(-\sqrt{3}, \sqrt{3})}$

$f(\sqrt{3}) = (\sqrt{3})^3 - 9\sqrt{3}$

$\sqrt{9}\sqrt{3}$

$3\sqrt{3} - 9\sqrt{3} = -6\sqrt{3}$

$\boxed{(\sqrt{3}, -6\sqrt{3})}$

36

$f(x) = x^3 - 9y$

$f'(x) = 3x^2 - 9$

$3x^2 - 9 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(3^{\frac{1}{2}})^3 - 9(3)^{\frac{1}{2}}$

$3^{\frac{3}{2}} - 3^2 3^{\frac{1}{2}}$

$3^{\frac{3}{2}} - 3^{2\frac{1}{2}}$

$3^{\frac{3}{2}} - 3^{\frac{5}{2}} = 3^{-\frac{2}{2}}$

$\frac{1}{3}$

$\boxed{\left(\sqrt{3}, \frac{1}{3}\right)}$

$(-\sqrt{3})^{\frac{3}{2}} - 9(-\sqrt{3})$

DNE $\sqrt{-\sqrt{3}}$

37 $h'(x) = 5f'(x) - \frac{2}{3}g'(x)$

$h'(4) = 5f'(4) - \frac{2}{3}g'(4)$

$5 \cdot 3 - \frac{2}{3} \cdot 4$

$15 - \frac{8}{3} \to \frac{45}{3} - \frac{8}{3} \to \frac{37}{3}$

38 $h(x) = x^2 f(x)$ product:

$h'(x) = f(x)(2x) + x^2 f'(x)$

$h'(4) = f(4)(8) + 16f'(4)$

$-8(8) + 16(3)$

$-64 + 48 = -16$

39 $h(x) = f(x)g(x)$

$h'(x) = g(x) \cdot f'(x) + f(x) \cdot g'(x)$

$h'(4) = g(4) \cdot f'(4) + f(4) \cdot g'(4)$

$3\pi \cdot 3 + -8 \cdot 4$

$\boxed{9\pi - 32}$

40 $h(x) = \frac{g(x)}{f(x)} \to h'(x) = \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}$

$h'(4) = \frac{f(4)g'(4) - g(4)f'(4)}{(f(4))^2} = \frac{-8 \cdot 4 - 3\pi(3)}{(-8)^2}$

$\frac{-32 - 9\pi}{64}$

41

$$h(x) = \frac{f(x) + g(x)}{x}$$

$$h'(x) = \frac{x\left[f'(x) + g'(x)\right] - \left[f(x) + g(x)\right]1}{x^2}$$

$$h'(4) = \frac{4\left[3 + 4\right] - \left[-8 + 3\pi\right]1}{16}$$

$$\frac{28 + 8 - 3\pi}{16} \rightarrow \boxed{\frac{36 - 3\pi}{16}}$$

42

$$\frac{x^3}{2x} - \frac{3x^3}{2x} - \frac{4x}{2x} - \frac{1}{2x}$$

$$\frac{1}{2}x^2 - \frac{3}{2}x - 2x - \frac{1}{2} \cdot \frac{1}{x}$$

$$\frac{1}{2}x^2 - \frac{3}{2}x - 2x - \frac{1}{2}x^{-1}$$

$$f'(x) = x - \frac{3}{2} - 2 + \frac{1}{2}x^{-2}$$

$$f''(x) = 1 \cancel{+} x^{-3} \rightarrow 1 - \frac{1}{x^3}$$

(43) $f(x) = \dfrac{x}{x-4}$

We are stuck using quotient rule twice

$$f'(x) = \frac{(x-4)\,1 - x(1)}{(x-4)^2}$$

$$\frac{x-4-x}{(x-4)^2} = \frac{-4}{(x-4)^2} \to \frac{-4}{x^2-8x+16}$$

$$f''(x) = \frac{(x^2-8x+16)(0)-(-4)(2x-8)}{(x-4)^4}$$

$$\boxed{\frac{8x-32}{(x-4)^4}}$$

44 $f(x) = x^{\frac{1}{2}} - 4x^{\frac{1}{3}} + 6x^{-\frac{1}{4}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{4}{3}x^{-\frac{2}{3}} - \frac{3}{2}x^{-\frac{5}{4}}$

$$\boxed{f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{8}{9}x^{-\frac{5}{3}} + \frac{15}{8}x^{-\frac{9}{4}}}$$

45 $f(x) = 2x^2 - 6x + 7$

$y = 2x + 4$

$f'(x) = 4x - 6$ $\quad f'(x) = 2$

$4x - 6 = 2$

$4x = 8$

$x = 2$

$f(2) = 2(4) - 12 + 7$

$8 - 12 + 7$

$15 - 12$

3

~~2,2~~ (2,3) ~~8~~

Slope 2

$y - 3 = 2(x - 3)$ $\quad \parallel$

$y - 3 = -\frac{1}{2}(x - 3)$ $\quad \perp$