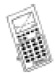
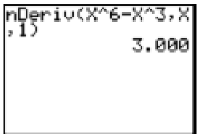
7. Derivatives Using Technology - Homework

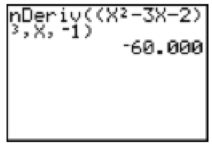


For each function $f(x)$, find $f'(a)$ using the calculator.

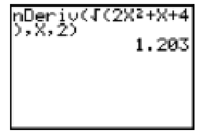
1. $f(x)=x^6-x^3,\ a=1$



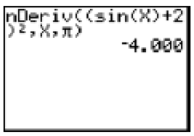
2. $f(x)=\left(x^2-3x-2\right)^3,\ a=-1$



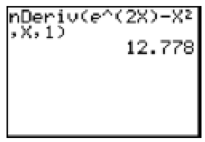
3. $f(x)=\sqrt{2x^2+x+4},\ a=2$



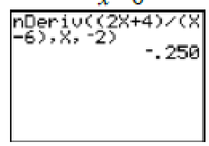
4. $f(x)=(\sin x+2)^2,\ a=\pi$



5. $f(x)=e^{2x}-x^2,\ a=1$

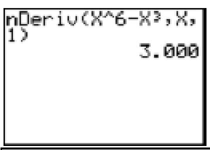


6. $f(x)=\dfrac{2x+4}{x-6},a=-2$



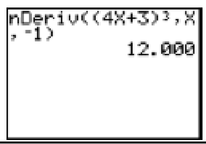
For each function $f(x)$, find the equation of the tangent line at $x=a$ using the calculator.

7. $f(x)=x^6-x^3,\ a=1$



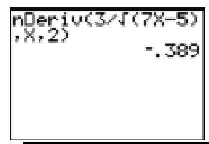
$y=3(x-1)\Rightarrow y=3x-3$

8. $f(x)=(4x+3)^3,\ a=-1$



$y+1=12(x+1)\Rightarrow y=12x+11$

9. $f(x)=\dfrac{3}{\sqrt{7x-5}},\ a=2$

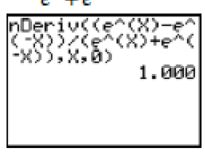


$y-1=-0.389(x-2)$

10. $f(x)=\sin x\cos x,\ a=\dfrac{\pi}{4}$

$y=\dfrac{1}{2}$

11. $f(x)=\dfrac{e^x-e^{-x}}{e^x+e^{-x}},\ a=0$



$y=x$

12. $f(x)=\sin(\ln(4x-1)),\ a=\dfrac{1}{2}$

$y=4\left(x-\dfrac{1}{2}\right)\Rightarrow y=4x-2$

Show the following using your calculator.

8. Techniques of Differentiation - Homework

For the following functions, find $f'(x)$ and $f'(c)$ at the indicated value of c. Use proper notation.

1. $f(x)=x^2-6x+1,\ c=0$

$$f'(x)=2x-6$$
$$f'(0)=-6$$

2. $f(x)=\frac{1}{x}-\frac{3}{x^2}+\frac{4}{x^3},\ c=-1$

$$f'(x)=\frac{-1}{x^2}+\frac{6}{x^3}-\frac{12}{x^4}$$
$$f'(-1)=-1-6-12=-19$$

3. $f(x)=3\sqrt{x}-\frac{1}{\sqrt[3]{x}},\ c=1$

$$f'(x)=\frac{3}{2x^{1/2}}+\frac{1}{3x^{4/3}}$$
$$f'(1)=\frac{3}{2}+\frac{1}{3}=\frac{11}{6}$$

For the following functions, find the derivative using the power rule. Use proper notation.

4. $y=\pi^5$

$$y'=0$$

5. $y=\frac{x^2+2}{6}$

$$y'=\frac{x}{3}$$

6. $y=\frac{1}{a}\left(x^2-\frac{1}{b^2}x+c\right)$, a,b,c constants

$$y'=\frac{1}{a}\left(2x-\frac{1}{b^2}\right)$$

7. $y=\frac{8}{3x^2}$

$$y=\frac{8}{3}x^{-2}$$
$$y'=\frac{-16}{3}x^{-3}=\frac{-16}{3x^3}$$

8. $y=\frac{-9}{\left(3x^2\right)^3}$

$$y=\frac{-9}{27x^6}=\frac{-1}{3}x^{-6}$$
$$y'=2x^{-7}=\frac{2}{x^7}$$

9. $y=\frac{6x^{3/2}}{x}$

$$y=6x^{1/2}$$
$$y'=3x^{-1/2}=\frac{3}{x^{1/2}}$$

10. $y=\frac{4x^2-5x+6}{3}$

$$y'=\frac{1}{3}(8x-5)$$

11. $y=\frac{x^2-6x+2}{2x}$

$$y=\frac{x}{2}-3+\frac{1}{x}$$
$$y'=\frac{1}{2}-\frac{1}{x^2}$$

12. $y=\frac{x^3+8}{x+2}$

$$y=\frac{(x+2)\left(x^2-2x+4\right)}{x+2}$$
$$y'=2x-2$$

13. $y=x^4-\frac{3}{2}x^3+5x^2-6x-2$

$$y'=4x^3-\frac{9}{2}x^2+10x-6$$

14. $y=\frac{x^3-3x^2+10x-5}{x^2}$

$$y=x-3+\frac{10}{x}-\frac{5}{x^2}$$
$$y'=1-\frac{10}{x^2}+\frac{10}{x^3}$$

15. $y=\left(x^2+4x\right)(2x-1)$

$$y=2x^3+7x^2-4x$$
$$y'=6x^2+14x-4$$

16. $y=(x-2)^3$

$$y=x^3-6x^2+12x-8$$
$$y'=3x^2-12x+12$$

17. $y=\sqrt[3]{x}-\sqrt[3]{x^2}$

$$y=x^{1/3}-x^{2/3}$$
$$y'=\frac{1}{3x^{2/3}}-\frac{2}{3x^{1/3}}$$

18. $y=\dfrac{x^4-2x^3+5x^2-4x+4}{x}$

$$y=x^3-2x^2+5x-4+\frac{4}{x}$$
$$y'=3x^2-4x+5-\frac{4}{x^2}$$

For the following functions, find the derivatives. Simplify all but # 19 fully. Use proper notation.

19. $y=(x^2-4x-6)(x^3-5x^2-3x)$

$$\frac{dy}{dx}=(x^2-4x-6)(3x^2-10x-3)+(x^3-5x^2-3x)(2x-4)$$

20. $y=\dfrac{3x-2}{2x+3}$

$$\frac{dy}{dx}=\frac{3(2x+3)-2(3x-2)}{(2x+3)^2}$$
$$\frac{dy}{dx}=\frac{13}{(2x+3)^2}$$

21. $y=\dfrac{x^2-4x-2}{x^2-1}$

$$\frac{dy}{dx}=\frac{(x^2-1)(2x-4)-(x^2-4x-2)(2x)}{(x^2-1)^2}$$
$$\frac{dy}{dx}=\frac{2x^3-4x^2-2x+4-2x^3+8x^2+4x}{(x^2-1)^2}$$
$$\frac{dy}{dx}=\frac{4x^2+2x+4}{(x^2-1)^2}$$

22. $y=\dfrac{x-1}{\sqrt{x}}$

$$y=x^{1/2}-x^{-1/2}$$
$$\frac{dy}{dx}=\frac{1}{2x^{1/2}}+\frac{1}{2x^{3/2}}$$

23. $y=\dfrac{x^2-x+1}{\sqrt[3]{x}}$

$$y=x^{5/3}-x^{2/3}+x^{-1/3}$$
$$\frac{dy}{dx}=\frac{5}{3}x^{2/3}-\frac{2}{3x^{1/3}}-\frac{1}{3x^{4/3}}$$

24. $y=\left(\dfrac{x-3}{x-4}\right)(3x+2)$

$$y=\frac{3x^2-7x-6}{x-4}$$
$$\frac{dy}{dx}=\frac{(x-4)(6x-7)-(3x^2-7x-6)}{(x-4)^2}$$
$$\frac{dy}{dx}=\frac{3x^2-24x+34}{(x-4)^2}$$

25. $y=\dfrac{1}{(x^2-5)^2}$

$$y'=\frac{-(4x^3-20x)}{(x^4-10x^2+25)^2}$$
$$y'=\frac{-4x(x^2-5)}{(x^2-5)^4}=\frac{-4x}{(x^2-5)^3}$$

26. $y=\dfrac{x+k}{x-k}$, k is a constant

$$\frac{dy}{dx}=\frac{(x-k)-(x+k)}{(x-k)^2}$$
$$\frac{dy}{dx}=\frac{-2k}{(x-k)^2}$$

27. $y=\dfrac{x^2+k^2}{x^2-k^2}$, k is a constant

$$\frac{dy}{dx}=\frac{(x^2-k^2)2x-(x^2+k^2)2x}{(x^2-k^2)^2}$$
$$\frac{dy}{dx}=\frac{-4k^2x}{(x^2-k^2)^2}$$

Find the equation of the tangent line to the graph of *f* at the indicated point and then use your calculator to confirm the results.

28. $f(x)=\dfrac{x^2}{x-1}$ at $(2,4)$

$f'(x)=\dfrac{(x-1)2x-x^2}{(x-1)^2}=\dfrac{x^2-2x}{(x-1)^2}$

$f'(2)=0$ Tan line: $y-4=0(x-2)\Rightarrow y=4$

29. $f(x)=(x-2)(x^2-3x-1)$ at $(-1,-9)$

$f'(x)=(x-2)(2x-3)+x^2-3x-1$

$f'(-1)=-3(-5)+1+3-1=18$

Tan line: $y+9=18(x+1)\Rightarrow y=18x+9$

30. $f(x)=\dfrac{x^2-4x+2}{2x-1}$ at $\left(2,\dfrac{-2}{3}\right)$

$f'(x)=\dfrac{(2x-1)(2x-4)-2(x^2-4x+2)}{(2x-1)^2}$

$f'(2)=\dfrac{3(0)-2(-2)}{9}=\dfrac{4}{9}$

Tan line: $y+\dfrac{2}{3}=\dfrac{4}{9}(x-2)\Rightarrow y=\dfrac{4}{9}x-\dfrac{14}{9}$

31. $f(x)=\dfrac{-1}{(2x-3)^2}$ at $\left(0,-\dfrac{1}{9}\right)$

$f'(x)=\dfrac{8x-12}{(4x^2-12x+9)^2}$

$f'(0)=\dfrac{-12}{81}=\dfrac{-4}{27}$

Tan line: $y+\dfrac{1}{9}=\dfrac{-4}{27}x\Rightarrow y=\dfrac{-4x-3}{27}$

Find the equation of the normal line to the graph of *f* at the indicated point and then use your calculator to confirm the results.

32. $f(x)=4x^2-8x-3$ at $\left(\dfrac{1}{2},-6\right)$

$f'(x)=8x-8$

$f'\left(\dfrac{1}{2}\right)=4-8=-4 \quad m\perp=\dfrac{1}{4}$

Normal line: $y+6=\dfrac{1}{4}\left(x-\dfrac{1}{2}\right)$

33. $f(x)=\dfrac{\sqrt{x}+4}{\sqrt{x}}$ at $(4,3)$

$f(x)=1+4x^{-1/2}\Rightarrow f'(x)=-2x^{-3/2}$

$f'(4)=\dfrac{-2}{8}=\dfrac{-1}{4} \quad m\perp=4$

Normal line: $y-3=4(x-4)\Rightarrow y=4x-13$

Determine the point(s) at which the graphs of the following functions have a horizontal tangent.

34. $y=\dfrac{x^2}{x^2-4}$

$f'(x)=\dfrac{(x^2-4)2x-x^2(2x)}{(x^2-4)^2}$

$\dfrac{-8x}{(x^2-4)^2}=0$

$x=0$ Pt: $(0,0)$

35. $f(x)=\dfrac{4x}{x^2+4}$

$f'(x)=\dfrac{(x^2+4)4-4x(2x)}{(x^2+4)^2}$

$\dfrac{-4(x^2-4)}{(x^2+4)^2}=\dfrac{-4(x+2)(x-2)}{(x^2+4)^2}=0$

$x=\pm 2$ Pts: $(2,1),(-2,-1)$

36. $f(x)=x^3-9x$

$f'(x)=3x^2-9=0$

$3(x^2-3)=0\Rightarrow x=\pm\sqrt{3}$

Pts: $(\sqrt{3},-6\sqrt{3}),(-\sqrt{3},6\sqrt{3})$

Use the chart to find $h'(4)$.

$f(4)$	$f'(4)$	$g(4)$	$g'(4)$
-8	3	3π	4

37. $h(x)=5f(x)-\frac{2}{3}g(x)$

$$h'(x)=5f'(x)-\frac{2}{3}g'(x)$$
$$h'(4)=5f'(4)-\frac{2}{3}g'(4)$$
$$h'(4)=5(3)-\frac{2}{3}(4)=\frac{37}{3}$$

38. $h(x)=x^2f(x)$

$$h'(x)=x^2f'(x)+2x\cdot f(x)$$
$$h'(4)=16f'(4)+8f(4)$$
$$h'(4)=16(3)+8(-8)=-16$$

39. $h(x)=f(x)g(x)$

$$h'(x)=f(x)g'(x)+g(x)f'(x)$$
$$h'(4)=f(4)g'(4)+g(4)f'(4)$$
$$h'(4)=-8(4)+3\pi(3)=9\pi-32$$

40. $h(x)=\frac{g(x)}{f(x)}$

$$h'(x)=\frac{f(x)g'(x)-g(x)f'(x)}{[f(x)]^2}$$
$$h'(4)=\frac{f(4)g'(4)-g(4)f'(4)}{[f(4)]^2}$$
$$h'(4)=\frac{-8(4)-3\pi(3)}{(-8)^2}=\frac{-9\pi-32}{64}$$

41. $h(x)=\frac{f(x)+g(x)}{x}$

$$h'(x)=\frac{x[f'(x)+g'(x)]-[f(x)+g(x)]}{x^2}$$
$$h'(4)=\frac{4[f'(4)+g'(4)]-[f(x)+g(4)]}{4^2}$$
$$h'(4)=\frac{4(3+4)-(-8+3\pi)}{16}=\frac{36-3\pi}{16}$$

For each of the following, find $f''(x)$.

42. $f(x)=\frac{x^3-3x^2-4x-1}{2x}$

$$f(x)=\frac{1}{2}\left(x^2-3x-4-\frac{1}{x}\right)$$
$$f'(x)=\frac{1}{2}\left(2x-3+\frac{1}{x^2}\right)$$
$$f''(x)=\frac{1}{2}\left(2-\frac{2}{x^3}\right)=1-\frac{1}{x^3}$$

43. $f(x)=\frac{x}{x-4}$

$$f'(x)=\frac{x-4-x}{(x-4)^2}=\frac{-4}{x^2-8x+16}$$
$$f''(x)=\frac{4(2x-8)}{(x^2-8x+16)^2}$$
$$f''(x)=\frac{8(x-4)}{(x-4)^4}=\frac{8}{(x-4)^3}$$

44. $f(x)=\sqrt{x}-4\sqrt[3]{x}+\frac{6}{\sqrt[4]{x}}$

$$f(x)=x^{1/2}-4x^{1/3}+6x^{-1/4}$$
$$f'(x)=\frac{1}{2}x^{-1/2}-\frac{4}{3}x^{-2/3}-\frac{3}{2}x^{-5/4}$$
$$f''(x)=\frac{-1}{4}x^{-3/2}+\frac{8}{9}x^{-5/3}+\frac{15}{8}x^{-9/4}$$
$$f''(x)=\frac{-1}{4x^{3/2}}+\frac{8}{9x^{5/3}}+\frac{15}{8x^{9/4}}$$

45. Find an equation of the line tangent to $f(x)=x^2-6x+7$ and

a. parallel to the line $y=2x+4$

parallel means same slope so slope of $f(x)=$ slope of line

$f'(x)=2x-6=2\Rightarrow 2x=8\Rightarrow x=4$

the point of tangency is $(4,f(4))=(4,-1)$

Parallel line: $y+1=2(x-4)\Rightarrow y=2x-9$

b. perpendicular to the line $y=2x+4$

perpendicular means slope of $f(x)=\frac{-1}{\text{slope of line}}$

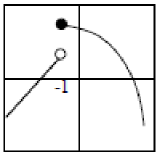
$f'(x)=2x-6=\frac{-1}{2}\Rightarrow 2x=\frac{11}{2}\Rightarrow x=\frac{11}{4}$

point of tangency is $\left(\frac{11}{4},f\left(\frac{11}{4}\right)\right)=\left(\frac{11}{4},\frac{-31}{16}\right)$

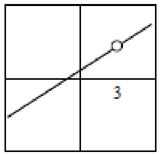
Perpendicular line: $y+\frac{31}{16}=\frac{-1}{2}\left(x-\frac{11}{4}\right)$

15. Continuity and Differentiability – Homework

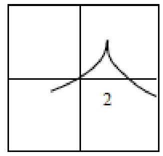
1. For the following graphs, determine if the function $f(x)$ is continuous at the marked value of *c*, and if not, determine for which of the 3 rules of continuity the function fails.

a. 

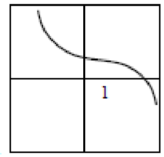
$\lim_{x \to -1} f(x)$ DNE

b. 

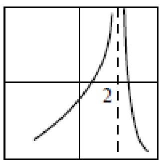
$f(3)$ DNE

c. 

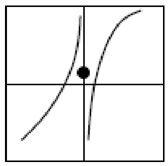
continuous

d. 

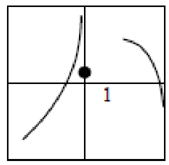
continuous

e. 

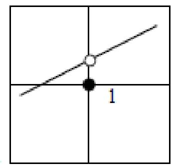
$\lim_{x \to 2} f(x)$, $f(2)$ DNE

f. 

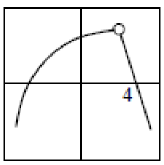
$\lim_{x \to 0} f(x)$ DNE

g. 

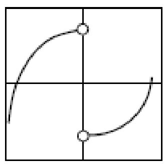
$f(1)$ DNE

h. 

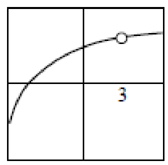
$\lim_{x \to 0} f(x) \neq f(0)$

i. 

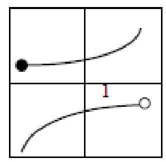
$f(4)$ DNE

j. 

$\lim_{x \to 0} f(x), f(0)$ DNE

k. 

$f(3)$ DNE

l. 

not a function

2. Find the value of *x* where the function is discontinuous.

a. $f(x) = x^3 + 3^x$

continuous everywhere

b. $f(x) = \dfrac{x}{x^2 - 81}$

$x = \pm 9$

c.[15a] $f(x) = \dfrac{x^2 - 36}{x^2 + 2x - 24}$

$x = -6, x = 4$

d. $f(x) = \tan x$

$x = \dfrac{n\pi}{2}, n$ an odd integer

3. Determine whether the function is continuous at the value where the rule changes. If not, explain why.

a. $f(x) = \begin{cases} 8 - x^2, x < 2 \\ 6 - x, x \geq 2 \end{cases}$

continuous

b. $f(x) = \begin{cases} 4 - x^2, x < 1 \\ 1 + x, x \geq 1 \end{cases}$

not continuous
$\lim_{x \to 1^-} f(x) = 3,\ \lim_{x \to 1^+} f(x) = 2$

c. $f(x) = \begin{cases} 2^x, x < 3 \\ \sqrt{7x^2 - 1}, x > 3 \end{cases}$

not continuous
$f(3)$ DNE

d. $f(x)=\begin{cases}2^{-x}, x<-1\\ x+3, x\geq -1\end{cases}$

e. $f(x)=\begin{cases}\dfrac{1}{x-2}, x<2\\ 3, x=2\\ x+1, x>2\end{cases}$

f. $f(x)=\begin{cases}\dfrac{x^3-x}{x^2-x}, x\neq 0, x\neq 1\\ 3, x=0\\ 2, x=1\end{cases}$

continuous

$\lim_{x\to 2^-} f(x)$ DNE

cont. at $x=1$, not cont. at $x=0$
$\lim_{x\to 0} f(x)=1\neq f(0)$

4. Find the value of the constant a that makes the function continuous.

a. $f(x)=\begin{cases}0.4x+2, x<1\\ 0.3x+a, x\geq 1\end{cases}$

b. $f(x)=\begin{cases}x^2, x<2\\ a-x, x\geq 2\end{cases}$

c. $f(x)=\begin{cases}9-x^2, x<2\\ ax, x\geq 2\end{cases}$

$0.4+2=0.3+a$
$a=2.1$

$4=a-2$
$a=6$

$9-4=2a$
$a=\dfrac{5}{2}$

d. $f(x)=\begin{cases}a-e^{-x}, x<0\\ x-a, x\geq 0\end{cases}$

e. $f(x)=\begin{cases}a^2+4x, x<-1\\ ax+16, x\geq -1\end{cases}$

f. $f(x)=\begin{cases}x\cos(\pi x), x<2\\ e^{ax}, x\geq 2\end{cases}$

$a-1=-a$
$a=\dfrac{1}{2}$

$a^2-4=-a+16$
$(a+5)(a-4)=0$
$a=-5, a=4$

$2=e^{2a}$
$2a=\ln 2$
$a=\dfrac{\ln 2}{2}$

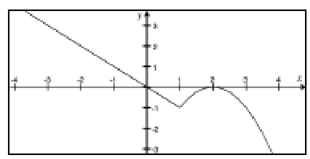
5. Let a and b represent constants and let $f(x)=\begin{cases}b-x, x<1\\ a(x-2)^2, x\geq 1\end{cases}$

a. Find an equation relating a and b if f is continuous at $x=1$.

b. Find b if $a = -1$. Graph the function.

c. Find other values of a, b where f is continuous

a. $a=b-1$ b. 0 c. $a=3, b=4$



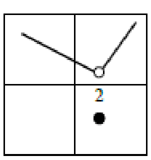
6. Sketch functions having the following attributes.

a. has a value of $f(2)$, $\lim_{x\to 2} f(x)$ exists but is not continuous at $x=2$.

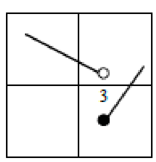
b. has a step discontinuity at $x=3$.

c. $\lim_{x\to 4} f(x)=0$ but $f(x)$ is not continuous at $x=4$.

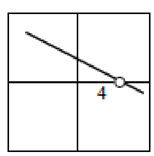
d. the value of $f(-2)=0$, and $\lim_{x\to -2^+} f(x)=-\lim_{x\to -2^-} f(x)$ and $f(x)$ is not continuous at $x=-2$.



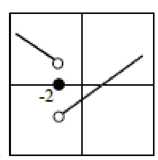
a.



b.

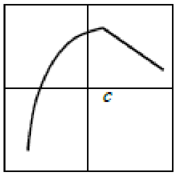


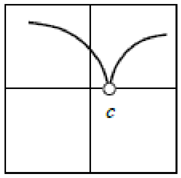
c.

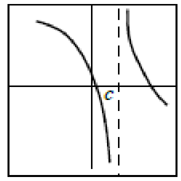


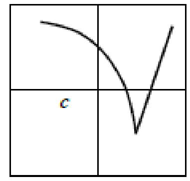
d.

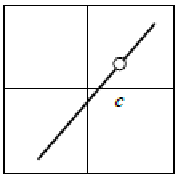
7. For the following, state whether the function is continuous, differentiable, both, or neither at $x = c$.

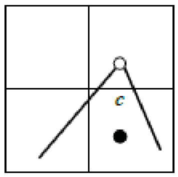
a. 
continuous only

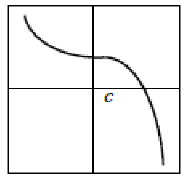
b. 
neither

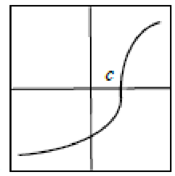
c. 
neither

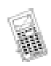
d. 
both

e. 
neither

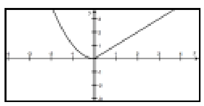
f. 
neither

g. 
both

h. 
continuous only

8. For the following functions, show work to determine whether the function is continuous, differentiable, both, or neither and sketch the curve. 

a. $f(x)=\begin{cases} x^2, & x\le 0 \\ x, & x>0 \end{cases}$



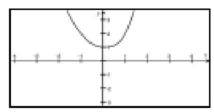
$\lim_{x\to 0^-} f(x)=0 \quad \lim_{x\to 0^+} f(x)=0 \Rightarrow \lim_{x\to 0} f(x)=0$

$f'(x)=\begin{cases} 2x, & x\le 0 \\ 1, & x>0 \end{cases}$

$\lim_{x\to 0^-} f'(x)=0 \quad \lim_{x\to 0^+} f'(x)=1 \Rightarrow \lim_{x\to 0} f'(x)$ DNE

continuous only

b. $f(x)=\begin{cases} x^2+1, & x\le 0 \\ x^3+1, & x>0 \end{cases}$



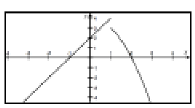
$\lim_{x\to 0^-} f(x)=1 \quad \lim_{x\to 0^+} f(x)=1 \Rightarrow \lim_{x\to 0} f(x)=1$

$f'(x)=\begin{cases} 2x, & x\le 0 \\ 3x^2, & x>0 \end{cases}$

$\lim_{x\to 0^-} f'(x)=0 \quad \lim_{x\to 0^+} f'(x)=0 \Rightarrow \lim_{x\to 0} f'(x)=0$

continuous and differentiable

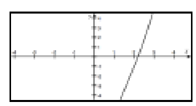
c. $f(x)=\begin{cases} 2x+2, & x\le 1 \\ 4-x^2, & x>1 \end{cases}$



$\lim_{x\to 1^-} f(x)=4 \quad \lim_{x\to 1^+} f(x)=3 \Rightarrow \lim_{x\to 1} f(x)$ DNE

Neither continuous nor differentiable

d. $f(x)=\begin{cases} 5x-11, & x\le 2 \\ x^2+x-7, & x>2 \end{cases}$



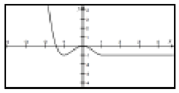
$\lim_{x\to 2^-} f(x)=-1 \quad \lim_{x\to 2^+} f(x)=-1 \Rightarrow \lim_{x\to 2} f(x)=-1$

$f'(x)=\begin{cases} 5, & x\le 2 \\ 2x+1, & x>2 \end{cases}$

$\lim_{x\to 2^-} f'(x)=5 \quad \lim_{x\to 2^+} f'(x)=5 \Rightarrow \lim_{x\to 2} f'(x)=5$

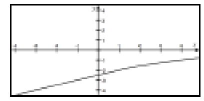
continuous and differentiable

e. $f(x)=\begin{cases} x^4-2x^2, \ x\le 1 \\ -1, \ x>1 \end{cases}$



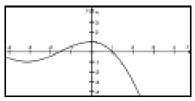
$$\lim_{x\to 1^-} f(x)=-1 \quad \lim_{x\to 1^+} f(x)=-1 \Rightarrow \lim_{x\to 1} f(x)=-1$$
$$f'(x)=\begin{cases} 4x^3-4x, \ x\le 1 \\ 0, \ x>1 \end{cases}$$
$$\lim_{x\to 1^-} f'(x)=0 \quad \lim_{x\to 1^+} f'(x)=0 \Rightarrow \lim_{x\to 1} f'(x)=0$$
continuous and differentiable

f. $f(x)=\begin{cases} \dfrac{x-5}{2}, \ x\le 1 \\ \sqrt{x}-3, \ x>1 \end{cases}$



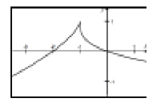
$$\lim_{x\to 1^-} f(x)=-2 \quad \lim_{x\to 1^+} f(x)=-2 \Rightarrow \lim_{x\to 1} f(x)=-2$$
$$f'(x)=\begin{cases} \dfrac{1}{2}, \ x\le 1 \\ \dfrac{1}{2\sqrt{x}}, \ x>1 \end{cases}$$
$$\lim_{x\to 1^-} f'(x)=\frac{1}{2} \quad \lim_{x\to 1^+} f'(x)=\frac{1}{2} \Rightarrow \lim_{x\to 1} f'(x)=\frac{1}{2}$$
continuous and differentiable

g. $f(x)=\begin{cases} \cos x, \ x\le 0 \\ 1-x^2, \ x>0 \end{cases}$



$$\lim_{x\to 0^-} f(x)=1 \quad \lim_{x\to 0^+} f(x)=1 \Rightarrow \lim_{x\to 0} f(x)=1$$
$$f'(x)=\begin{cases} -\sin x, \ x\le 0 \\ -2x, \ x>0 \end{cases}$$
$$\lim_{x\to 0^-} f'(x)=0 \quad \lim_{x\to 0^+} f'(x)=0 \Rightarrow \lim_{x\to 0} f'(x)=0$$
continuous and differentiable

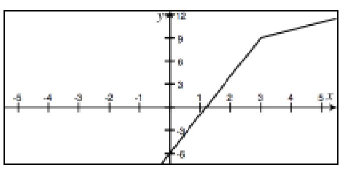
h. $f(x)=\begin{cases} 1-\sqrt[3]{(x+1)^2}, \ x\le -1 \\ 1-\sqrt[3]{x+1}, \ x>-1 \end{cases}$



$$\lim_{x\to -1^-} f(x)=1 \quad \lim_{x\to -1^+} f(x)=1 \Rightarrow \lim_{x\to -1} f(x)=1$$
$$f'(x)=\begin{cases} \dfrac{-2}{3\sqrt[3]{x+1}}, \ x\le -1 \\ \dfrac{-1}{3\sqrt[3]{(x+1)^2}} \ x>-1 \end{cases}$$
$$\lim_{x\to -1^-} f'(x)=\infty \quad \lim_{x\to -1^+} f'(x)=-\infty \Rightarrow \lim_{x\to -1} f'(x) \text{DNE}$$
continuous only

9. Given the graph of $f(x)$ and its equation, give a calculus reason for whether or not $f(x)$ is differentiable.

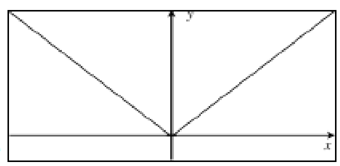
a.



$f(x)=3x-|2x-6|$

$$f(x)=\begin{cases} 3x-(2x-6), x\ge 3 \\ 3x+(2x-6), x<3 \end{cases} \qquad f'(x)=\begin{cases} 1, x\ge 3 \\ 5, x<3 \end{cases}$$
$$\lim_{x\to 3^+} f'(x)=1 \quad \lim_{x\to 3^-} f'(x)=5 \Rightarrow \lim_{x\to 3} f'(x) \text{ DNE}$$
Therefore, $f(x)$ is not differentiable

b. 15b



$f(x)=\sin\sqrt{x^2+0.000001}-0.001$

$$f'(x)=\cos\sqrt{x^2+0.000001}\left(\frac{2x}{2\sqrt{x^2+0.000001}}\right)$$
$f'(x)=0$ at $x=0$ so $f(x)$ is differentiable.
Zooming into the origin will show that this graph does not have a corner. This problem shows that graphical explanations are not sufficient.

a. $f(x)=\begin{cases} x^3, & x\le 1 \\ a(x-2)^2+b, & x>1 \end{cases}$

$\lim\limits_{x\to 1^-} f(x)=1 \quad \lim\limits_{x\to 1^+} f(x)=a+b$

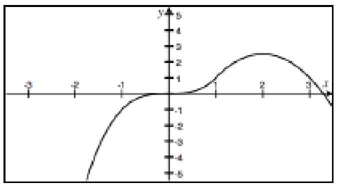
To be continuous, $a+b=1$

$f'(x)=\begin{cases} 3x^2, & x\le 1 \\ 2a(x-2), & x>1 \end{cases}$

$\lim\limits_{x\to 1^-} f'(x)=3 \quad \lim\limits_{x\to 1^+} f'(x)=-2a$

To be differentiable, $-2a=3$

$a=\frac{-3}{2},\ b=\frac{5}{2}$



b. $f(x)=\begin{cases} ax^2+10, & x\le 2 \\ x^2-6x+b, & x>2 \end{cases}$

$\lim\limits_{x\to 2^-} f(x)=4a+10 \quad \lim\limits_{x\to 2^+} f(x)=-8+b$

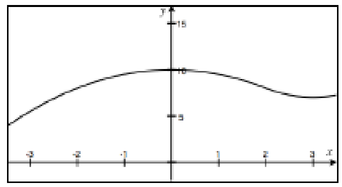
To be continuous, $4a+10=-8+b \Rightarrow 4a=b-18$

$f'(x)=\begin{cases} 2ax, & x\le 2 \\ 2x-6, & x>2 \end{cases}$

$\lim\limits_{x\to 2^-} f'(x)=4a \quad \lim\limits_{x\to 2^+} f'(x)=-2$

To be differentiable, $4a=-2$

$a=\frac{-1}{2},\ b=16$



c. $f(x)=\begin{cases} 12-bx^2, & x\le 1 \\ \frac{a}{x}, & x>1 \end{cases}$

$\lim\limits_{x\to 1^-} f(x)=12-b \quad \lim\limits_{x\to 1^+} f(x)=a$

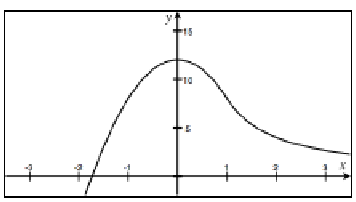
To be continuous, $12-b=a$

$f'(x)=\begin{cases} -2bx, & x\le 1 \\ \frac{-a}{x^2}, & x>1 \end{cases}$

$\lim\limits_{x\to 1^-} f'(x)=-2b \quad \lim\limits_{x\to 1^+} f'(x)=-a$

To be differentiable, $-2b=-a$

$a=8,\ b=4$



d. $f(x)=\begin{cases} 3a+be^{-x}+1, & x\le 0 \\ a(\sin x+\cos x)+bx, & x>0 \end{cases}$

$\lim\limits_{x\to 0^-} f(x)=3a+b+1 \quad \lim\limits_{x\to 0^+} f(x)=a$

To be continuous, $3a+b+1=a \Rightarrow 2a+b=-1$

$f'(x)=\begin{cases} -be^{-x}, & x\le 0 \\ a(\cos x-\sin x)+b, & x>0 \end{cases}$

$\lim\limits_{x\to 0^-} f'(x)=-b \quad \lim\limits_{x\to 0^+} f'(x)=a+b$

To be differentiable, $-b=a+b \to a+2b=0$

$a=\frac{-2}{3}, b=\frac{1}{3}$