Central Limit Theorem:

21. Gestation Period The length of human pregnancies is approximately normally distributed with mean $\mu = 266$ days and standard deviation $\sigma = 16$ days.

NW

(a) What is the probability a randomly selected pregnancy lasts less than 260 days?

(b) Suppose a random sample of 20 pregnancies is obtained. Describe the sampling distribution of the sample mean length of human pregnancies.

(c) What is the probability that a random sample of 20 pregnancies has a mean gestation period of 260 days or less?

(d) What is the probability that a random sample of 50 pregnancies has a mean gestation period of 260 days or less?

(e) What might you conclude if a random sample of 50 pregnancies resulted in a mean gestation period of 260 days or less?

(f) What is the probability a random sample of size 15 will have a mean gestation period within 10 days of the mean?

22. Upper Leg Length The upper leg length of 20- to 29-year-old males is normally distributed with a mean length of 43.7 cm and a standard deviation of 4.2 cm.

Source: "Anthropometric Reference Data for Children and Adults: U.S. Population, 1999–2002; Volume 361, July 7, 2005.

(a) What is the probability that a randomly selected 20- to 29-year-old male has an upper leg length that is less than 40 cm?

(b) A random sample of 9 males who are 20 to 29 years old is obtained. What is the probability that the mean upper leg length is less than 40 cm?

(c) What is the probability that a random sample of 12 males who are 20 to 29 years old results in a mean upper leg length that is less than 40 cm?

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

(e) A random sample of 15 males who are 20 to 29 years old results in a mean upper leg length of 46 cm. Do you find this result unusual? Why?

23. Reading Rates The reading speed of second grade students is approximately normal, with a mean of 90 words per minute (wpm) and a standard deviation of 10 wpm.

(a) What is the probability a randomly selected student will read more than 95 words per minute?

(b) What is the probability that a random sample of 12 second grade students results in a mean reading rate of more than 95 words per minute?

(c) What is the probability that a random sample of 24 second grade students results in a mean reading rate of more than 95 words per minute?

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

(e) A teacher instituted a new reading program at school. After 10 weeks in the program, it was found that the mean reading speed of a random sample of 20 second grade students was 92.8 wpm. What might you conclude based on this result?

24. Old Faithful The most famous geyser in the world, Old Faithful in Yellowstone National Park, has a mean time between eruptions of 85 minutes. If the interval of time between eruptions is normally distributed with standard deviation 21.25 minutes, answer the following questions:
Source: www.unmuseum.org

(a) What is the probability that a randomly selected time interval between eruptions is longer than 95 minutes?

(b) What is the probability that a random sample of 20 time intervals between eruptions has a mean longer than 95 minutes?

(c) What is the probability that a random sample of 30 time intervals between eruptions has a mean longer than 95 minutes?

(d) What effect does increasing the sample size have on the probability? Provide an explanation for this result.

(e) What might you conclude if a random sample of 30 time intervals between eruptions has a mean longer than 95 minutes?

25. Rates of Return in Stocks The S&P 500 is a collection of 500 stocks of publicly traded companies. Using data obtained from Yahoo!Finance, the monthly rates of return of the S&P 500 since 1950 are normally distributed. The mean rate of return is 0.007233 (0.7233%), and the standard deviation for rate of return is 0.04135 (4.135%).

(a) What is the probability that a randomly selected month has a positive rate of return? That is, what is $P(x > 0)$?

(b) Treating the next 12 months as a simple random sample, what is the probability that the mean monthly rate of return will be positive? That is, with $n = 12$, what is $P(\overline{x} > 0)$?

(c) Treating the next 24 months as a simple random sample, what is the probability that the mean monthly rate of return will be positive?

(d) Treating the next 36 months as a simple random sample, what is the probability that the mean monthly rate of return will be positive?

(e) Use the results of parts (b)–(d) to describe the likelihood of earning a positive rate of return on stocks as the investment time horizon increases.

29. Insect Fragments The Food and Drug Administration sets Food Defect Action Levels (FDALs) for some of the various foreign substances that inevitably end up in the food we eat and liquids we drink. For example, the FDAL for insect filth in peanut butter is 3 insect fragments (larvae, eggs, body parts, and so on) per 10 grams. A random sample of 50 ten-gram portions of peanut butter is obtained and results in a sample mean of $\overline{x} = 3.6$ insect fragments per ten-gram portion.

(a) Why is the sampling distribution of $\overline{x}$ approximately normal?

(b) What is the mean and standard deviation of the sampling distribution of $\overline{x}$ assuming that $\mu = 3$ and $\sigma = \sqrt{3}$.

(c) What is the probability that a simple random sample of 50 ten-gram portions results in a mean of at least 3.6 insect fragments? Is this result unusual? What might we conclude?

31. Watching Television The amount of time Americans spend watching television is closely monitored by firms such as A. C. Nielsen because this helps to determine advertising pricing for commercials.

(a) Do you think the variable "weekly time spent watching television" would be normally distributed? If not, what shape would you expect the variable to have?

(b) According to the American Time Use Survey, adult Americans spend 2.35 hours per day watching television on a weekday. Assume that the standard deviation for "time spent watching television on a weekday" is 1.93 hours. If a random sample of 40 adult Americans is obtained, describe the sampling distribution of $\overline{x}$, the mean amount of time spent watching television on a weekday.

(c) Determine the probability that a random sample of 40 adult Americans results in a mean time watching television on a weekday of between 2 and 3 hours.

(d) One consequence of the popularity of the Internet is that it is thought to reduce television watching. Suppose that a random sample of 35 individuals who consider themselves to be avid Internet users results in a mean time of 1.89 hours watching television on a weekday. Determine the likelihood of obtaining a sample mean of 1.89 hours or less from a population whose mean is presumed to be 2.35 hours. Based on the result obtained, do you think avid Internet users watch less television?

32. ATM Withdrawals According to ATMDepot.com, the mean ATM withdrawal is \$60. Assume that the standard deviation for withdrawals is \$35.

(a) Do you think the variable "ATM withdrawal" is normally distributed? If not, what shape would you expect the variable to have?

(b) If a random sample of 50 ATM withdrawals is obtained, describe the sampling distribution of $\overline{x}$, the mean withdrawal amount.

(c) Determine the probability of obtaining a sample mean withdrawal amount between \$70 and \$75.

Normal Approximation for the Binomial Distribution

22. Smokers According to *Information Please Almanac*, 80% of adult smokers started smoking before they were 18 years old. Suppose 100 smokers 18 years old or older, are randomly selected. Use the normal approximation to the binomial to

(a) approximate the probability that exactly 80 of them started smoking before they were 18 years old.

(b) approximate the probability that at least 80 of them started smoking before they were 18 years old.

(c) approximate the probability that fewer than 70 of them started smoking before they were 18 years old.

(d) approximate the probability that between 70 and 90 of them, inclusive, started smoking before they were 18 years old.

23. Migraine Sufferers In clinical trials of a medication whose purpose is to reduce the pain associated with migraine headaches, 2% of the patients in the study experienced weight gain as a side effect. A random sample of 600 users of this medication is obtained. Use the normal approximation to the binomial to

(a) approximate the probability that exactly 20 will experience weight gain as a side effect.
(b) approximate the probability that 20 or fewer will experience weight gain as a side effect.
(c) approximate the probability that 22 or more patients will experience weight gain as a side effect.
(d) approximate the probability that between 20 and 30 patients, inclusive, will experience weight gain as a side effect.

24. Murder by Firearms According to the *Uniform Crime Report, 2005*, 67.8% of murders are committed with a firearm. Suppose that 50 murders are randomly selected. Use the normal approximation to the binomial to

(a) approximate the probability that exactly 40 murders are committed using a firearm.
(b) approximate the probability that at least 35 murders are committed using a firearm.
(c) approximate the probability that fewer than 25 murders are committed using a firearm.
(d) approximate the probability that between 30 and 35 murders, inclusive, are committed using a firearm.

25. Males Living at Home According to the *Current Population Survey* (Internet release date: September 15, 2004), 55% of males between the ages of 18 and 24 years lived at home in

2003. (Unmarried college students living in a dorm are counted as living at home.) A survey is administered at a community college to 200 randomly selected male students between the ages of 18 and 24 years, and 130 of them respond that they live at home.

(a) Approximate the probability that such a survey will result in at least 130 of the respondents living at home under the assumption that the true percentage is 55%.

(b) Does the result from part (a) contradict the results of the *Current Population Survey*? Explain.

26. Females Living at Home According to the *Current Population Survey* (Internet release date: September 15, 2004), 46% of females between the ages of 18 and 24 years lived at home in 2003. (Unmarried college students living in a dorm are counted as living at home.) A survey is administered at a community college to 200 randomly selected female students between the ages of 18 and 24 years, and 110 of them respond that they live at home.

(a) Approximate the probability that such a survey will result in at least 110 of the respondents living at home under the assumption that the true percentage is 46%.

(b) Does the result from part (a) contradict the results of the *Current Population Survey*? Explain.

27. Boys Are Preferred In a Gallup poll conducted June 11–14, 2007, 37% of survey respondents said that, if they only had one child, they would prefer the child to be a boy. You conduct a survey of 150 randomly selected students on your campus and find that 80 of them would prefer a boy.

NW

(a) Approximate the probability that, in a random sample of 150 students, at least 75 would prefer a boy, assuming the true percentage is 37%.

(b) Does this result contradict the Gallup poll? Explain.

28. Liars According to a *USA Today* "Snapshot," 3% of Americans surveyed lie frequently. You conduct a survey of 500 college students and find that 20 of them lie frequently.

(a) Compute the probability that, in a random sample of 500 college students, at least 20 lie frequently, assuming the true percentage is 3%.

(b) Does this result contradict the *USA Today* "Snapshot"? Explain.