Chapter 4: Problem Set

Numbers with an asterisk* have solutions in the back of the book.

Basics (4.1)

1.* **Boys and Girls:** A couple plans to have two children. Assume the probability of a girl is 0.50.

   (a) What is the sample space for the gender outcomes in having two children.
   (b) What is the probability that the couple has one boy and one girl?
   (c) What is the probability that the couple will have at least one girl?
   (d) What is the probability that the couple will have no girls?

2. **Cards:** Suppose you draw one card from a single deck of cards. A deck has 52 cards consisting of 13 hearts, 13 clubs, 13 spades, and 13 diamonds.

   (a) What is the probability that you draw a king?
   (b) What is the probability that you draw a heart?
   (c) What is the probability that you draw the king of hearts?

3.* **Lottery:** You enter a lottery by purchasing one of the 1,000 tickets. There is one grand prize winner, 2 second prize winners, and 5 small prize winners. These are selected at random (without replacement) from a bin containing all the tickets.

   (a) What is the probability that you will win the grand prize?
   (b) What is the probability that you will win a prize of some type?
   (c) If the winners are chosen with replacement, the problem becomes a lot more complicated. Why is that?

4. **Same Birthday:** If two people are randomly selected from a class of 30 students, what is the probability that they have the same birthday?

5.* **Car Accidents:** Of the reported 1567 car accidents in Stevens County last year, 923 of them occurred within 1 mile of the person's home.

   (a) If one of the reported car accidents is randomly selected, what is the probability that it occurred within 1 mile of the person's home?
   (b) Estimate the probability that the next reported car accident in Stevens County will be within 1 mile of the person's home?

6. **Life Insurance:** A life insurance company wants to estimate the probability that a 40 year old male will live through the next year. In a random sample of 80,000 such men, 79,968 lived through the year. Using the relative frequency approximation, what is the probability that a randomly selected 40 year old male will die within a year?

7.* **Warranty:** In setting the warranty price for MP3 players, an electronics company wants to estimate the probability that a randomly purchased MP3 player will fail within the year. In 4000 randomly sold MP3 players, 24 of them failed within the year. Using the relative frequency approximation, what is the probability that the next MP3 player sold will fail within a year?

8. **Odd or Even:** You ask 30 people to choose a number between 1 and 10, and 22 of them choose an odd number. Estimate the probability that the next person you ask will choose an odd number?

9.* **Weather Forecast:** The table below indicates the accuracy of a local weather report with respect to rain or no rain over the past year. This table gives the results of 365 consecutive days and compares whether it rained or not to whether or not rain was predicted.

	Did it actually rain? Yes	No
Report Predicted Rain	102	18
Report Predicted No Rain	40	205

(a) How many times was the prediction correct? How many times was it wrong.

(b) If one day is randomly selected from last year, what is the probability that the prediction was correct?

(c) Tomorrow, the same local weather report will be given. Estimate the probability that it will be correct with respect to rain or no rain?

Conditional Probabilities(4.2)

10.* **Weather Forecast:** The table below indicates the accuracy of a local weather report with respect to rain or no rain over the past year. This table gives the results of 365 consecutive days and compares whether it rained or not to whether or not rain was predicted.

	Did it actually rain? Yes	No
Report Predicted Rain	102	18
Report Predicted No Rain	40	205

If one day is randomly selected from these 365 days, what is the probability that

(a) the prediction was correct.

(b) it rained given that it was predicted to rain (This is called true-positive).

(c) it did not rain when it was predicted not to rain. (This is called a true-negative).

(d) Based on these results is this weather forecast better at predicting rain or better at predicting no rain?

11. **Pregnancy Test:** A pregnancy testing device is used by 1000 different women from a population of women who think they might be pregnant. The results are depicted in the contingency table below. Here, a positive test result means pregnancy is detected.

	Was the woman actually pregnant?	
	Yes	No
positive test result	479	13
negative test result	6	502

(a) Using the relative frequency approximation of probabilities, what is the probability that the device is correct?

(b) Suppose you are a woman about to take the test. Prior to taking the test, what is the probability of a false-positive?

(c) Suppose you are a woman who takes the test and it comes back positive. Now, what is the probability that the test result is wrong?

(d) Comment on the difference between your answer to part (b) and your answer to part (c).

12. **Cards:** Suppose you and a friend are playing cards and you are each dealt 4 cards. You have a 10, Jack, Queen, and King in your *hand*. You are about to be dealt one more card. What is the probability that you are dealt an Ace given that

(a)* Your friend has no aces in his hand.

(b) Your friend has exactly one ace in his hand.

13. **Cards:** Suppose you are playing Poker alone. You have four cards (3♡, 4♡, 5♡, and 6♡). You are about to select one more card from the remaining deck. What is the probability that you get

(a)* a flush (all cards of the same suit)?

(b)* a straight (5 consecutive cards)?

(c) a straight flush (5 consecutive cards of the same suit)?

The Addition Rule (4.3)

14. **Mutually Exclusive Events:** Determine whether the events are mutually exclusive or not.

(a)* Rolling a single die and getting a 6. Rolling a single die and getting a 2.

(b) Randomly selecting a person with brown eyes. Randomly selecting a person with red hair.

(c) Randomly selecting a person with brown eyes. Randomly selecting a person with blue eyes.

(d)* Ordering a meal with vegetables. Ordering a vegetarian meal.

15.* **Dice:** Suppose you roll two dice - a red one and a white one. There are 36 different outcomes in this sample space (for each of the 6 options on the red die, there are 6 options for the white one). What is the probability that

(a) the total of the dice is 3?

(b) the total of the dice is not 3?

16.* **Ski Passes:** The following table gives some information about a group of 200 College students.

	Has a ski pass at				
	Bolton	Stowe	Smuggler's	Sugarbush	No Pass
Has a Car	18	12	10	30	25
Does not have a Car	24	5	4	12	60

(a) Do any of these 200 students have a pass to more than one mountain?

(b) If one student is randomly selected, what is the probability of selecting a person with a pass at Stowe or Sugarbush?

(c) If one student is randomly selected, what is the probability of selecting a person with a car or a pass to Sugarbush?

(d) If one student is randomly selected, what is the probability of selecting a person with a car or a ski pass? **Hint:** It might be easier to calculate the probability of selecting someone not in either category.

17. **Blood Types:** The following table summarizes blood types for 100 typical people. For example, a person with type O$^+$ blood actually has group O and type Rh^+ blood.

	Group			
	O	A	B	AB
Type Rh^+	37	36	9	3
Type Rh^-	7	6	1	1

If one person is randomly selected, find the probability of

(a) Selecting a person who is Group O or type Rh^+.

(b) Selecting a person who is Group A or Group B

The Multiplication Rule (4.4)

18. **Independent Events:** For each of the following examples, determine if the two events are dependent or independent.

    (a)* Drawing a Jack on the first of two draws and drawing a Jack on the second of two draws without replacement.

    (b)* Purchasing one gallon of spoiled milk from the store and purchasing another gallon of spoiled milk from the same store.

    (c)* Flipping a fair coin twice and getting *heads* on the first toss and getting *heads* on the second toss.

    (d) Winning the lottery on Tuesday and winning the lottery on the following Wednesday.

    (e) Finding that your microwave doesn't work and finding that your refrigerator doesn't work.

    (f) Finding that your microwave doesn't work and finding that your left shoe is untied.

19.* **Cards:** Suppose you draw two cards **with replacement** from a standard deck.

    (a) What is the probability of getting a Jack then a Jack again?

    (b) What is the probability of getting a Jack then a Queen?

    (c) What is the probability of getting a Jack then a Heart?

20. **Cards:** Suppose you draw two cards **without replacement** from a standard deck.

    (a) What is the probability of getting a Jack then a Jack again?

    (b) What is the probability of getting a Jack then a Queen?

21. **Extended Multiplication Rule:** Use the extended multiplication rule to calculate the following probabilities.

    (a) If you flip a coin 4 times, what is the probability of getting 4 heads.

    (b)* If you randomly select 4 people, what is the probability that they were born on the same day of the week (Monday, Tuesday, ... Sunday) ?

    (c) Assume that your car starts 99% of the time. What is the probability that your car will start for the next 14 days in a row.

22. **Marbles:** Suppose a box contains 12 marbles, 3 are red, 4 are blue, and 5 are yellow. Find the requested probabilities.

    (a)* If two marbles are selected **with replacement**, what is the probability that one is red and the other is yellow.

    (b) If two marbles are selected **without replacement**, what is the probability that one is red and one is yellow.

    (c)* If 5 marbles are selected **without replacement**, what is the probability that they are all yellow.

23. **Senators:** Use the following data for the 100 Senators from the 112^{th} Congress of the United States.

	Republican	Democrat	Independent
Male	42	39	2
Female	5	12	0

(a)* If two different senators are randomly selected, what is the probability that they are both male republicans?

(b)* If two different senators are randomly selected, what is the probability that the first is a democrat and the second is a republican?

(c) If two different senators are randomly selected, what is the probability that they are both female democrats?

(d) If two different senators are randomly selected, what is the probability that the first is a female and the second is a male?

One Bad Apple - Probabilities of At Least One (4.5)

24.* **Spark Plugs:** Assume that 2% of all spark-plugs are defective.

(a) If you buy one spark plug, what is the probability that it is **not** defective?

(b) If you buy 4 spark plugs, what is the probability that all 4 are **not** defective?

(c) If you buy 4 spark plugs, what is the probability that at least one is defective?

25. **At Least One Girl:** Suppose a couple plans to have 4 children and the probability of a boy is 0.50. Find the probability that the couple has at least one girl.

26.* **Lie Detector:** Suppose a lie detector test can detect a lie 95% of the time. You get hooked up and tell 10 truths and 10 lies. What is the probability that at least one of your lies goes undetected?

27. **Alarm Clock - Redundancy:** You have two alarm clocks. The first one is successful 95% of the time and the second one is successful 60% of the time (it turns out your second one was actually less reliable than the first).

(a) Suppose you only remember to set the good alarm clock. What is the probability that it will succeed on the morning of an important exam?

(b) Suppose you set both alarm clocks. What is the probability that at least one of them is successful on the morning of an important exam?

(c) This practice of using a second device is called **redundancy**. Was there a significant increase in the probability of getting to the exam obtained using the second alarm clock?

28.* **Jumper-Cables:** Assume that 25% of all car owners have jumper-cables in the car. You are stranded in a parking lot with a dead battery and there are 3 other people getting into different cars nearby.

(a) What is the probability that all three people have jumper-cables in the car?

(b) What is the probability that at least one of the three people have jumper-cables in the car?

(c) Which probability is more relevant to your current situation?

29. **Smoke Detectors:** Suppose a certain brand of smoke detector functions properly 85% of the time. Now to be extra safe, I have installed 3 of these smoke detectors in my bedroom. Calculate the following probabilities **rounding your answer to 3 significant digits**.

(a) What is the probability that all three smoke detectors function properly on the night of a fire?

(b) What is the probability that at least one of the three smoke detectors functions properly on the night of a fire?

(c) Comment on the difference in the probabilities found in parts (a) and (b). Which probability is more relevant to the situation?