Chapter 5: Problem Set

Numbers with an asterisk* have solutions in the back of the book.

Discrete Random Variable and Probability Distributions (5.1)

1.* **Coin Flips:** The table below gives the probability distribution for the number of heads in four tosses of a fair coin.

# of heads	$P(x)$
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

(a) Verify that this is indeed a probability distribution.

(b) What is the mean of the probability distribution.

(c) If you flip a coin 4 times, what is the expected value for the number of heads you get?

2. **Roll of a Die:** Suppose you roll a single six-sided die with numbers 1 - 6 printed on the sides. Assume that each side has an equal probability of being rolled.

(a) Create a probability distribution for the number showing on one roll of a die.

(b) Calculate the mean of this probability distribution.

(c) If you were to roll such a die over and over while recording the number on the face each time, what do expect would be the mean from all these numbers?

3.* **Lottery:** I buy one of 200 raffle tickets for $10. The sponsors then randomly select one grand prize worth $200, two second prizes worth $100 each, and three third prizes at $50 each. Below is the discrete probability distribution for this raffle.

Prize	$P(x)$
Grand	1/200
Second	2/200
Third	3/200
None	194/200

(a) Verify that this is a probability distribution.

(b) Recognizing that I spent $ 10 to buy a ticket, determine the expected value of this raffle to me as a player.

4. **Lottery:** I buy one of 5000 raffle tickets for \$1. The sponsors then randomly select one grand prize worth \$500, two second prizes worth \$200 each, and three third prizes at \$100 each. Create the probability distribution for this raffle and calculate my expected value.

5.* **Warranty:** Suppose you buy a \$150 cell phone. You do not buy the \$10 replacement warranty but will buy another one at full price if it fails. Suppose there is a 5% chance that it will fail. Based on expected cost, did you make the right decision? What does the \$10 warranty represent?

6. **Life Insurance:** Your company sells life insurance. You charge a 30 year old man \$25 for a one year, \$100,000 policy. If he dies over the course of the next year you pay out \$100,000. If he lives, you keep the \$25. Based on historical data (relative frequency approximation) the average 30 year old man has a 0.9999 probability of living through the year.

   (a) What is your expected profit on this policy?

   (b) What is the *break-even* price of such a policy? I.e. What price should you charge to produce an expected profit of zero?

Binomial Distributions (5.2)

7.* Determine whether the following sequence of trials would result in a binomial probability distribution.

   (a) Calling 500 people and asking who they voted for in an election.

   (b) Calling all your friends until you have 2 people willing to play on your volleyball team.

8. Determine whether the following sequence of trials would result in a binomial probability distribution.

   (a) Calling 500 people and ask if they voted for a particular candidate in a given election.

   (b) The National Health Institute checks 100 people who had a certain type of cancer in the year 2000 and records whether they are alive or not.

9.* Calculate the following binomial probabilities by either using one of the binomial probability tables, or calculating the probability with a calculator or software using the formula

$$P(x|n,p) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x} \qquad \text{where } q = 1-p$$

   (a) $P(x = 6, n = 10, p = .8)$

   (b) $P(x = 15, n = 17, p = .8)$

   (c) $P(x < 4, n = 15, p = .2)$

   (d) $P(x \geq 2, n = 8, p = .4)$

10. Calculate the following binomial probabilities by either using one of the binomial probability tables, or calculating the probability with a calculator or software using the formula

$$P(x|n,p) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x} \qquad \text{where } q = 1 - p$$

    (a) $P(x = 4, n = 15, p = .2)$

    (b) $P(x = 9, n = 12, p = .75)$

    (c) $P(x > 6, n = 10, p = .8)$

    (d) $P(x < 20, n = 20, p = .9)$

11.* **Cards:** Suppose you draw a card from a deck (with replacement) 10 times in a row. What is the probability that you get exactly 4 hearts?

12.* **Lie Detector:** Suppose a lie detector allows 20% of all lies to go undetected. If you take the test and tell 10 lies, would 5 undetected lies be an unusually large number of undetected lies? Use the criteria that a number (x) is unusually large if $P(x \text{ or more}) \leq 0.05$.

13. **T/F Quiz:** Suppose you take a 10 question True or False quiz and you guess on every problem. You only get 2 correct. Is this an unusually low number of correct guesses. Use the criteria that a number (x) is unusually low if $P(x \text{ or less}) \leq 0.05$.

14.* **Over-booking Flights:** At Low Budget Air (LBA), historical data shows that 95% of all passengers *show up* in time to board. Therefore, they occasionally over-book flights in hopes of filling each plane to capacity. Here we consider some probabilities associated with this type of issue. Suppose a small plane has a capacity of 20 passengers. They book 20 passengers for a flight on this plane.

    (a) What is the probability that exactly 20 passengers *show up*?
    (b) What is the probability that exactly 19 passengers *show up*?
    (c) What is the probability that they will have at least one empty seat?
    (d) If only 17 passengers *show up*, would that be considered an unusually low number of passengers? Use the criteria that a number (x) is unusually low if $P(x \text{ or less}) \leq 0.05$.
    (e) If only 16 passengers *show up*, would that be considered an unusually low number of passengers? Use the criteria that a number (x) is unusually low if $P(x \text{ or less}) \leq 0.05$.

Mean and Standard Deviation of Binomial Distributions (5.3)

Be sure to verify that $n \cdot p \geq 5$ and $n \cdot q \geq 5$ when doing these problems.

15.* **Cancer Survival Rates:** Suppose a certain type of cancer has a 0.75 survival rate for five years. This means that 75% of those that got this type of cancer did not die from it five years later. Now, suppose you check on 130 people with this type of cancer 5 years after diagnosis.

(a) In such groups of 130 what is the mean number of survivors at the five-year mark?

(b) What is the standard deviation?

(c) If you check on 130 such patients from a certain hospital and find that only 85 survived, would you categorize this as unusual?

16. **Uninsured:** It is estimated that 16.6% of all adults in the U.S. are uninsured. You take a random sample of 250 adults seen by a certain clinic and find that 50 are uninsured.

(a) In such groups of 250 U.S adults, what is the mean number of those that would be uninsured?

(b) What is the standard deviation?

(c) In your survey you found 50 of the 250 U.S. adults are uninsured. Would you categorize this number as unusual?

17.* **Good-Buy Electronics:** You own a branch of *Good-Buy* Electronics and have been told by the manufacturer of *Stevuski* Televisions that only 5% of their brand of TV's die within one year. Your branch sold 124 such televisions last month during a sale and 16 of them had been returned - dead.

(a) Assuming the 5% value quoted by the manufacturer was accurate what is the mean number of TV's that die within one year in randomly samples of size 124.

(b) Of the 124 TV's you sold, 16 of them died. Is this is unusual number?

(c) Name a couple of things that **could** have caused this unusual event.

18. **NHL - Birthdays:** It has been observed that a large percentage of National Hockey League (NHL) players have birthdays in the first part of the year. It has been suggested that this is due to the cut-off dates for participation in the youth leagues - those born in the earlier months are older than their peers and this advantage is amplified over the years via more opportunities to train and be coached. Of the 512 players in the 2008/2009 NHL season, 159 of them were born in January, February, or March.

(a) Assume that 25% of birthdays from the general population occur in January, February, or March (these actually contain 24.7% of the days of the year). In random samples of 512 people, what is the mean number of those with a birthday in January, February, or March?

(b) Recognizing that 159 of the 512 NHL players were born in the first three months of the year. Does this suggest that there is something unusual happening?