Chapter 6: Problem Set

Numbers with an asterisk* have solutions in the back of the book.

The Standard Normal Distribution (6.1)

1. Find the requested probabilities from the standard normal distribution (z-table).

   (a)* (i) $P(z < 1.96)$ (ii) $P(z \geq 2.13)$ (iii) $P(-1.21 < z < 2.13)$

   (b) (i) $P(z \leq -1.45)$ (ii) $P(z > -1.84)$ (iii) $P(0.35 < z < 2.13)$

2.* For these questions, z^* represents some z-score.

   (a) Suppose $P(z \leq z^*) = .85$. Give two reasons why $P(z > z^*) = 0.15$.
   Hint: One reason has to do with the area under a probability density curve, and the other is a concept discussed in the chapter on probability.

   (b) Suppose $P(z \leq z^*) = 0.85$. Is z^* positive or negative and why?

   (c) Suppose $P(z < z^*) = 0.85$. What is the $P(-z^* < z < z^*)$?

3. Here we find a z-score that corresponds to a given percentile

   (a)* Find the z-score that marks the 85^{th} percentile (denoted P_{85}). That is, find the z-score that delineates the lower 85% from the upper 15% of all the z-scores.

   (b) Find the z-score that marks the 25^{th} percentile (denoted P_{25}). That is, find the z-score that delineates the lower 25% from the upper 75% of all the z-scores.

4. Here we find the z-scores that delineate the middle portions of the standard normal distribution.

   (a)* Find the range of z-scores that constitute the middle 95% of all values from the others.

   (b) Find the range of z-scores that constitute the middle 99% of all values from the others.

Normal Distributions in General (6.2)

5.* **Potatoes:** Suppose the weights of Farmer Carl's potatoes are normally distributed with a mean of 8 ounces and a standard deviation of 1.2 ounces.

   (a) If one potato is randomly selected, find the probability that it weighs less than 10 ounces.

   (b) If one potato is randomly selected, find the probability that it weighs more than 12 ounces.

   (c) If one potato is randomly selected, find the probability that it weighs between 10 and 12 ounces.

   (d) Carl only wants to sell the best potatoes to his friends and neighbors at the farmer's market. According to weight, this means he wants to sell only those potatoes that are among the heaviest 20%. What is the minimum weight required to be brought to the farmer's market.

   (e) He wants to use the lightest potatoes as ammunition for his potato launcher but can only spare about 5% of his crop for such frivolities. What is the weight limit for potatoes to be considered for ammunition.

   (f) Determine the weights that delineate the middle 90% of Carl's potatoes from the others.

6. **Bass:** The bass in Clear Lake have weights that are normally distributed with a mean of 2.2 pounds and a standard deviation of 0.7 pounds.

   (a) If you catch one random bass from Clear Lake, find the probability that it weighs less than 1 pound.

   (b) If you catch one random bass from Clear Lake, find the probability that it weighs more than 3 pounds.

   (c) If you catch one random bass from Clear Lake, find the probability that it weighs between 1 and 3 pounds.

   (d) Suppose you only want to keep fish that are in the top 10% as far as weight is concerned. What is the minimum weight of a *keeper*?

   (e) Suppose you want to mount a fish if it is in the top 0.5% of those in the lake. What is the minimum weight of a bass to be mounted?

   (f) Determine the weights that delineate the middle 99% of the bass in Clear Lake.

7.* **Speeding:** On a certain stretch of Interstate-89, car speed is a normally distributed variable with a mean of 70 mph and a standard deviation of 4 mph.

   (a) You are traveling at 76 mph. Approximately what percentage of cars are traveling faster than you?

   (b) Suppose you are a police officer on this stretch of road and only have time to ticket 1 in 50 cars that go by you. How fast should someone be traveling before you pull them over?

8. **How Laude?** Many educational institutions award three levels of Latin honors often based on GPA. These are *cum laude* (with high praise), *magna cum laude* (with great praise), and *summa cum laude* (with highest praise). Requirements vary from school to school. Suppose the GPA's at State College are normally distributed with a mean of 2.85 and standard deviation of 0.45.

   (a) Suppose State College awards the top 2% of students (based on GPA) with the *summa cum laude* honor. What GPA gets you this honor?

   (b) Suppose State College awards the top 10% of students (based on GPA) with the *magna cum laude* honor. What GPA gets you this honor?

   (c) Suppose State College awards the top 20% of students (based on GPA) with the *cum laude* honor. What GPA gets you this honor?

Central Limit Theorem (6.4)

9.* **Potatoes - Samples:** Suppose the weights of Farmer Carl's potatoes are normally distributed with a mean of 8 ounces and a standard deviation of 1.2 ounces.

(a) If 4 potatoes are randomly selected, find the probability that the mean weight is less than 10 ounces.

(b) If 20 potatoes are randomly selected, find the probability that the mean weight is less than 10 ounces.

(c) Suppose Carl bags his potatoes in randomly selected groups of 6. What percentage of these bags should have a mean potato weight between 7.5 and 8.5 ounces?

(d) Suppose you buy a bag of Carl's potatoes at the Farmer's market. Each bag contains 6 potatoes. Your bag weighs 42 ounces. Do you feel cheated? How cheated?

10. **Bass - Samples:** The bass in Clear Lake have weights that are normally distributed with a mean of 2.2 pounds and a standard deviation of 0.7 pounds.

(a) If you catch 3 random bass from Clear Lake, find the probability that the mean weight is less than 1 pound.

(b) If you catch 3 random bass from Clear Lake, find the probability that the mean weight it is more than 3 pounds.

(c) What percentage of all randomly caught groups of 3 fish should weigh between 2.0 and 2.4 pounds?

(d) Suppose you have a *stringer* of 6 fish with a total weight of 16.2 pounds. Should you brag about this to your friends back at the lodge?

11.* **L.A. Lakers:** The 2009/2010 L.A. Lakers consists of 13 adult men. The mean height on this team was 79.0 inches and the mean weight was 228 pounds. Use the table below to answer the questions that follow it.

Strata	Mean Height (inches)	Standard Deviation Height (inches)	Mean Weight (pounds)	Standard Deviation Weight (pounds)
U.S. Men	69.3	2.8	191	28
NBA Players	79.0	2.1	221	25

(a) If 13 U.S. men are randomly selected, what is the probability that the mean height would be 79.0 inches or larger?

(b) If 13 U.S. men are randomly selected, what is the probability that the mean weight would be 228 pounds or more?

(c) If 13 NBA players are randomly selected, what is the probability that the mean height would be 79.0 inches or larger?

(d) If 13 NBA players are randomly selected, what is the probability that the mean weight would be 228 or larger?

(e) Does the roster of the L.A. Lakers seem to be unusually big and/or tall?

12. **Lifespan:** Assume the average life-span of those born in the U.S. is 78.2 years with a standard deviation of 16 years. The distribution is not normal (it is skewed left). The good people at *Live-Longer-USA* (fictitious) claim that their regiment of acorns and exercise results in longer life. So far, 40 people on this program have died and the mean age at death was 83.3 years.
    (a) Calculate the probability that a random sample of 40 from the general population would produce a mean age-of-death greater than 83.3 years.
    (b) Does this provide good evidence the the acorns and exercise program helps people live longer?
    (c) Why could we use the central limit theorem here despite the parent population being skewed?

The Normal Approximation to the Binomial Distribution (6.5)

13.* **Employee Satisfaction Rates:** A recent poll suggests that 48% of Americans are satisfied with their job. You have a company with 220 employees and a poll suggests that 85 of them are satisfied (quite a bit less than 48%). Is this an unusual number of satisfied employees? If so, how unusual?

14. **College Attendance:** About 68% of all U.S. public high school graduates in 2011 went on to attend college that fall. At Heavenly High, there were 200 graduates and 150 of them went on to attend college in the fall (75% of them). Did something special happen at Heavenly High with the graduating class of 2010? Should Heavenly High get the credit?

15.* **Uninsured Patients:** It is estimated that 16.6% of all adults in the U.S. are uninsured. You take a random sample of 250 adults seen by a certain clinic and find that 50 (20%) are uninsured. What is the probability of randomly selection 250 adults with 50 or more of them being uninsured. How unusual of an event is this?

16. **Pepperoni Appreciation:** Tony's Pizza Company finds that 65% of the general population likes pepperoni pizza. I buy pizza for 56 of my intro stats students and it turns out that only 20 of these students like pepperoni pizza. Is this an unusual sample of 56 people? How unusual? What could have caused such a low rate of pepperoni appreciation?

Others (time permitting)

17. **Empirical Rule Revisited:** In Chapter 2, you saw something called the *Empirical Rule* for data that is approximately normally distributed. It states that about 68% of all values fall within one standard deviation of the mean, 95% of all values fall within 2 standard deviations of the mean, and 99.7% of all values fall within 3 standard deviations of the mean. Check this rule for accuracy using the z-table.
    (a) For a normally distributed variable, find the probability that a value falls within one standard deviation of the mean.
    (b) For a normally distributed variable, find the probability that a value falls within two standard deviations of the mean.
    (c) For a normally distributed variable, find the probability that a value falls within three standard deviations of the mean.

18. **Light Bulbs:** The mean lifespan of a standard 60 watt incandescent light bulb is 875 hours with a standard deviation of 80 hours. The mean lifespan of a standard 14 watt compact fluorescent light bulb (CFL) is 10,000 hours with a standard deviation of 1,500 hours. These two bulbs put out about the same amount of light. Assume the lifespan's of both types of bulbs are normally distributed to answer the following questions.

    (a) I select one incandescent light bulb and put it in my barn. It seems to last forever and I estimate that it has lasted more than 2000 hours. What is the probability of selecting a random incandescent light bulb and having it last 2000 hours or more. Did something unusual happen here?

    (b) I select one CFL bulb and put it in the bathroom. It doesn't seem to last very long and I estimate that it has lasted less than 5,000 hours. What is the probability of selecting a random CFL and having it last less than 5,000 hours. Did something unusual happen here?

    (c) Compare the the lifespan of the middle 99% of all incandescent and CFL light bulbs.

    (d) Is there much of a chance that I happen to buy an incandescent light bulb that lasts longer than a randomly selected CFL?

19. **Hours Online:** The number of hours spent *online* by college students is claimed to be 22.5 hours per week with a standard deviation of 2.1 hours.

    (a) Suppose we randomly select 50 college students, what is the probability that the mean number of hours online is greater than 25.

    (b) Suppose I survey the 50 game programming students and the mean number of hours spent online is 25 hours per week. Is this unusual? Why or why not?

20.* **Birth weight:** A baby is said to have a low birth-weight when he or she weighs less than 5 pounds, 8 ounces (2,500 grams) and about 8.3% of all U.S. babies born fall into this category. Checking around it looks like the mean birth-weight for babies born in the U.S. is about 3,500 grams.

    (a) Assuming birth-weights are normally distributed, estimate the standard deviation of birth-weights given this information.

    (b) Suppose the U.S. Department of Health and Human Services wants to change the definition of **low birth-weight** to include only the lowest 5% of birth-weights. What would be the new definition of a low birth-weight baby? Use the standard deviation you calculated in part a).