Chapter 7: Problem Set

Numbers with an asterisk* have solutions in the back of the book.

Estimating a Population Mean and Sample Size (σ known) (7.2)

1.* **Sleep-College Students:** Suppose you perform a study about the hours of sleep that college students get. You know that for all people, the average is about 7 hours with a standard deviation of 1.25 hours. You randomly select 50 college students and survey them on their sleep habits. From this sample, the mean number of hours of sleep is found to be 6.20 hours. Assume the population standard deviation for college students is the same as for all people.

   (a) What is the point estimate for the mean amount of sleep for **all** college students?

   (b) Construct the 95% confidence interval for the mean number of hours of sleep for all college students.

   (c) Does your answer to the previous problem support the claim that college students get less than 6.5 hours of sleep on average.

   (d) Estimate the sample size required to be 90% confident that the sample mean is within 0.2 hours of the population mean for college students.

   (e) Estimate the sample size required to be 90% confident that the sample mean is within 0.1 hours of the population mean for college students.

   (f) Estimate the sample size required to be 99% confident that the sample mean is within 0.1 hours of the population mean for college students.

2. **Taste Buds - Top Chefs:** Assume the mean number of taste buds from the general population is 10,000 with a standard deviation of 850. You take a sample of 10 top chefs and find the mean number of taste buds is 11,500. Assume that the number of taste-buds is a normally distributed variable for top chefs and assume the standard deviation for top chefs is the same as for the general population.

   (a) What is the point estimate for the mean number of taste buds for **all** top chefs?

   (b) Construct the 90% confidence interval for the mean number of taste buds for all chefs.

   (c) Does your answer to the previous problem support the claim that top chefs have more taste buds, on average, than the general population.

   (d) Suppose you wanted to perform a similar study only this time you want a margin of error less than 300 at the 99% confidence level. How many top chefs would you have to sample?

   (e) Give two reasons why your answer to the previous problem was so much larger than the original sample size of 10.

3.* When constructing a confidence interval for a population mean with σ known, the margin of error is $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$. If you want a confidence interval at the 98% confidence level, what is the value for $z_{\alpha/2}$?

4. When constructing a confidence interval for a population mean with σ known, the margin of error is $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$. If you want a confidence interval at the 80% confidence level, what is the value for $z_{\alpha/2}$?

Estimating a Population Proportion and Sample Size (7.3)

5.* **Corn:** In a random sample of 80 ears of corn, farmer Carl finds that 12 of them have worms.

(a) What is the point estimate for the proportion of **all** of Carl's corn that has worms?

(b) What is the 95% confidence interval for the proportion of all of Carl's corn with worms?

(c) What is the 99% confidence interval for the proportion of all of Carl's corn with worms?

(d) Farmer Carl wants to be more accurate with his 99% confidence interval. He wants an estimate that is in error by no more than 2 percentage points. What sample size is needed to obtain this type of accuracy? Use his prior value of $\hat{p}$ to calculate this number.

(e) Repeat the previous problem only assume no knowledge of $\hat{p}$.

6. **Pinworm:** In a random sample of 800 adults in the U.S.A., it was found that 72 of those had a pinworm infestation.

(a) What is the point estimate for the proportion of **all** adults with pinworms?

(b) What is the 90% confidence interval for the proportion of all adults with pinworm?

(c) What is the 99% confidence interval for the proportion of all adults with pinworm?

(d) The Center for Disease Control (CDC) wants to improve the 99% confidence interval for the proportion of healthy adults with pinworm. They want an estimate that is in error by no more than one percentage point. What sample size is needed to obtain this type of accuracy? Use the prior value of $\hat{p}$ to calculate this number.

(e) Repeat the previous problem only assume no knowledge of $\hat{p}$.

(f) In Sludge County, the proportion of adults with pinworm is found to be 0.14. Based on your answer to (c), does Sludge County's pinworm infestation rate appear to be abnormal?

7.* **Fair Coin?** A coin is called *fair* if it lands on heads 50% of all possible flips. I flip a *Cheesy Chadwick's* game token 100 times and it comes up *heads* 59 times. I do not know whether this token is fair or not but I suspect it is not fair.

(a) What is the point estimate for the proportion of heads in **all** tosses of this token?

(b) What is the 99% confidence interval for the proportion of heads in **all** tosses of this token?

(c) Can you be 99% confident that the coin is **not** fair?

(d) What is the 90% confidence interval for the proportion of heads in **all** tosses of this token?

(e) Can you be 90% confident that the coin is **not** fair?

(f) Suppose I want to repeat my experiment with this token but I want a more accurate 99% confidence interval. How many times would I need to toss the coin to ensure that the margin of error is less than 0.04? Use the prior value of $\hat{p}$ to calculate this number.

(g) Repeat the previous problem only assume no knowledge of $\hat{p}$.

8. **Multiple Choice Strategy:** Some students have suggested that if you have to guess on a multiple-choice question, you should *always choose C*. Carl, the student, wants to investigate this theory. He is able to get a sample of past tests and quizzes from various teachers. In this sample there are 80 multiple-choice questions with four options (A,B,C,D). The distribution of correct answers from this sample is given in the frequency table to the right.

Correct Answer	Frequency
A	18
B	17
C	28
D	17

   (a) If the correct answers for all multiple-choice problems are uniformly distributed across the four options (A,B,C,D), what is the theoretical proportion of those which should have the answer C?

   (b) Based on the sample that Carl collected, what is the point estimate for the proportion of all multiple-choice questions with a correct answer of C?

   (c) What is the 90% confidence interval for the proportion of **all** multiple-choice questions with a correct answer of C?

   (d) Can Carl be 90% confident that the correct answer of C shows up more frequently than the theoretical value found in part (a) would suggest?

   (e) What is the 99% confidence interval for the proportion of **all** multiple-choice questions with a correct answer of C?

   (f) Can Carl be 99% confident that the correct answer of C shows up more frequently than the theoretical value found in part (a) would suggest?

Estimating a Population Mean (σ unknown) (7.4)

9.* **Student Debt - Vermont:** The average student loan debt of a U.S. college student at the end of 4 years of college is estimated to be about \$21,000. You take a random sample of 150 college students in the state of Vermont and find the mean debt is \$24,000 with a standard deviation of \$2,500.

   (a) Construct the 95% confidence interval for the mean student debt of all Vermont college graduates.

   (b) Based on the answer to (a), are you confident that the mean student debt of Vermont students is greater than the quoted national average of \$21,000?

10. **Sleep - College Students:** Suppose you perform a study about the hours of sleep that college students get. You know that for all people, the average is about 7 hours. You randomly select 50 college students and survey them on their sleep habits. From this sample, the mean number of hours of sleep is found to be 6.2 hours with a standard deviation of 0.75 hours.

   (a) What is the point estimate for the average number of hours of sleep for all college students?

   (b) Construct the 95% confidence interval for the mean number of hours of sleep for all college students.

   (c) Construct the 99% confidence interval for the mean number of hours of sleep for all college students.

   (d) Based on your prior answers are you confident that the mean number of hours slept by college students is less than the mean of 7 hours for the general population?

11.* **Salmon Weights:** Assume that the weights of spawning Chinook salmon in the Columbia River are normally distributed. You randomly catch and weigh 20 such salmon. The mean weight from your sample is 25.2 pounds with a standard deviation of 4.5 pounds.

(a) What is the point estimate for the mean weight of all spawning Chinook salmon in the Columbia River?

(b) Construct the 90% confidence interval for the mean weight of all spawning Chinook salmon in the Columbia River.

(c) Construct the 95% confidence interval for the mean weight of all spawning Chinook salmon in the Columbia River.

(d) Based on your prior answers are you confident that the mean weight of all spawning Chinook salmon in the Columbia River is greater than 23 pounds?

(e) Why were you able to use the t-distribution despite such a small sample?

12. **Diet Methods:** Here we study the effectiveness of two dieting methods. Method 1 is a diet high in protein, fiber, and fat but low in carbohydrates. Method 2 is a diet low in fat but high in fiber and carbohydrates. There were 60 participants for each method. After one year, Method 1 participants had a mean weight loss of 8.2 pounds with a standard deviation of 1.7 pounds. After one year, Method 2 participants had a mean weight loss of 7.5 pounds with a standard deviation of 1.2 pounds.

(a) Construct the 95% confidence interval for the mean weight loss after one year of all people who would complete the Method 1 diet.

(b) Construct the 95% confidence interval for the mean weight loss after one year of all people who would complete the Method 2 diet.

(c) Based on your answers above, does one method appear to be better than the other?

Summary Problems (7.5)

13.* **When to Use What:** For each example below you should start to construct a confidence interval by identifying which distribution should be used to find the margin of error. The options are the z-distribution, the t-distribution, or neither.

(a) A confidence interval for a population mean where $n = 20$, the population distribution is normal, $\bar{x} = 196.5$, σ is unknown, and $s = 12.56$.

(b) A a confidence interval for a population proportion where the sample size is 38 and the number of successes is 3.

(c) A confidence interval for a population mean where $n = 500$, the population distribution is normal, $\bar{x} = 0.087$, σ is known to be 0.003, and s is 0.0025.

(d) A confidence interval for a population proportion where the sample size is 22, and the number of successes is 10.

(e) A confidence interval for a population mean where $n = 20$, the population distribution is not normal, $\bar{x} = 130.7$, σ is unknown, and s=12.75.

14. **When to Use What:** For each example below you should start to construct a confidence interval by identifying which distribution should be used to find the margin of error. The options are the z-distribution, the t-distribution, or neither.

    (a) a confidence interval for a population mean where $n = 25$, the population distribution is skewed, $\bar{x} = 13.7$, and σ is known to be 2.5.

    (b) a confidence interval for a population proportion where the sample size is 250, and the number of successes is 113.

    (c) a confidence interval for a population mean where $n = 234$, the population distribution is skewed, $\bar{x} = 1435$, and σ is known to be 234.

    (d) a confidence interval for a population mean where $n = 15$, the population distribution is normal, $\bar{x} = 130.7$, σ is unknown, and s=12.75.

15.* **Finding $\overline{x}$ and E:** A 90% confidence interval for a population mean is given as $12.4 < \mu < 13.2$. Calculate the sample mean and the margin of error.

16. **Finding $\hat{p}$ and E:** A 95% confidence interval for a population proportion is given as $0.696 < p < 0.784$. Calculate the sample proportion and the margin of error.

17.* **Changes in the Margin of Error:** Suppose you are constructing confidence intervals about a mean or proportion. Answer the following questions about the margin of error (E)when the following changes are made.

    (a) If you increase the confidence level, what happens to the margin of error?

    (b) If you decrease the confidence level, what happens to the margin of error?

    (c) In general, what happens to the margin of error if the sample size is increased?

18. **Changes in the size of the Confidence Interval:** Suppose you are constructing confidence intervals about a mean or proportion. Answer the following questions about the size of the confidence interval when the following changes are made.

    (a) If you increase the confidence level, what happens to the size of the confidence interval?

    (b) If you decrease the confidence level, what happens to the size of the confidence interval?

    (c) In general, what happens to the size of the confidence interval when the sample size is increased?

19. **Common Misinterpretation:** Suppose a 90% confidence interval is given as $22.5 < \mu < 32.0$ and a class-mate says *this means that 90% of the data falls between the values 22.5 and 32.0*. What is wrong with this statement?