## Chapter 9: Problem Set

Numbers with an asterisk ${ }^{*}$ have solutions in the back of the book.

In some of the problems you are just given the necessary information (sample size, means, variances, standard deviations) to complete the test. These problems can be done by hand (with a calculator) or with software. In other problems you are given the raw data so that you can have software do all of the calculations. These problems also have the necessary information included in case you don't have sufficient software capabilities. For small sample sizes you may assume the parent populations are normally distributed.

## Mean Differences: Paired Data (9.1)

1. Sibling $I Q$ Scores: There have been numerous studies involving the correlation and differences in IQ's among siblings. Here we consider a small example of such a study. We will test the claim that older siblings have a higher IQ than younger siblings. The results are depicted for a sample of 10 brothers in the table below.

|  | IQ Score |  |  |  |  |  |  |  |  | mean | $s$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Older Brother $(x)$ | 84 | 87 | 91 | 92 | 99 | 104 | 105 | 110 | 114 | 121 | 100.7 | 12.2 |
| Younger Brother $(y)$ | 81 | 91 | 86 | 92 | 95 | 102 | 105 | 109 | 108 | 113 | 98.2 | 10.7 |
| $d=x-y$ | 3 | -4 | 5 | 0 | 4 | 2 | 0 | 1 | 6 | 8 | 2.5 | 3.5 |

(a) Test the claim at the 0.01 significance level.
(b) Test the claim at the 0.05 significance level.
(c) In an actual study involving IQ's of over 100,000 male siblings the mean difference was 2.3 in favor of the older siblings. Without having the actual data, speculate on whether you could support the claim at the 0.01 significance level.
(d) What other pattern is discernable just from the small sample data given?
2. Foot-Length: It is considered quite common to have feet of unequal length. In a sample of 10 healthy college students the right-foot and left-foot lengths are given (in mm).

|  | Length in mm |  |  |  |  |  |  |  |  | mean | $s$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Left Foot $(x)$ | 271 | 267 | 259 | 254 | 259 | 272 | 271 | 255 | 272 | 252 | 263.2 | 8.2 |
| Right Foot $(y)$ | 271 | 266 | 258 | 253 | 260 | 272 | 269 | 255 | 271 | 252 | 262.7 | 8.0 |
| $d=x-y$ | 0 | 1 | 1 | 1 | -1 | 0 | 2 | 0 | 1 | 0 | 0.5 | 0.8 |

(a) Test the claim that, on average, there is a measurable difference between left and right foot length. Do so at the 0.05 significance level.
(b) You should not have been able to support your claim from the previous problem. Does this mean it is unusual to have two feet of different length?
(c) How could you resolve the result of the hypothesis test with the idea that having a left/right foot discrepancy is not unusual? Can this be done with just the data given or do we have to start over with a larger sample?
3. Retaking the SAT: Many high school students take the SAT's twice; once in their Junior year and once in their Senior year. In a sample of 200 such students, the average of the differences was 32 points with a standard deviation of 14 points.
(a) Test the claim that retaking the SAT increases the score on average by more than 30 points. Use a 0.10 significance level.
(b) Can you support this claim at the 0.01 significance level?
4. AM -vs- PM Height: It is widely accepted that people are a little taller in the morning than at night. Here we perform a test on how big the difference is. In a sample of 35 adults, the mean difference between morning height and evening height was 5.8 millimeters ( mm ) with a standard deviation of 1.9 mm . Test the claim that, on average, people are more than 5 mm taller in the morning than at night. Test this claim at the 0.05 significance level.

## Two Means: Independent Data (9.2)

5. Math \& Music: There is a lot of interest in the relationship between studying music and studying math. We will look at some sample data that investigates this relationship. Here are the Math SAT scores from 8 students who studied music through high school and 11 students who did not. The degrees of freedom (d.f.) is given to save calculation time if you are not using software.

|  | MATH SAT Scores |  |  |  |  |  |  |  |  |  |  | mean | $s^{2}$ | $s$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Music ( $x_{1}$ ) | 552 | 605 | 596 | 650 | 556 | 555 | 585 | 634 |  |  |  | 591.6 | 1375.1 | 37.08 |
| No Music ( $x_{2}$ ) | 480 | 535 | 553 | 537 | 480 | 513 | 495 | 556 | 554 | 493 | 557 | 523.0 | 992.8 | 31.51 |
| degrees of freedom: d.f. $=14$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(a) Test the claim that students who study music in high school have a higher average Math SAT score than those who do not. Use a 0.01 significance level.
(b) (Software Required:) You should have been able to support the claim from part (a). So now the question is how much better do the music students do on the Math SAT scores. Via trial and error (with the aid of software), come up with a safe bet as to how much better students who study music do, on average, than those who do not study music.
6. Register Balance: Here we investigate whether the register balance at a local retail store is better on days with a manager than days without a manager. This evidence might be used to determine whether or not you should always schedule a manager. The table gives the register balance for a sample of 10 days with a manager and 10 days without a manager. Here, 0 means the register balance is right on, negative means there is less money than there should be, and positive means there is more money than there should be. The degrees of freedom (d.f.) is given in the table.

|  | Register Balance (10 days each) |  |  |  |  |  |  |  |  |  | mean | $s^{2}$ | $s$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| With Manager ( $x_{1}$ ) | -5 | 0 | -7 | -4 | 5 | -3 | -2 | -1 | -7 | -5 | -2.90 | 13.21 | 3.63 |
| Without Manager ( $x_{2}$ ) | 2 | -9 | -15 | -10 | -10 | 0 | -12 | -5 | 0 | -14 | -7.30 | 38.01 | 6.17 |
| degrees of freedom: d.f. $=15$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

(a) Test the claim that the average register balance is better (greater) for all days with a manager than those days without a manager. Use a 0.01 significance level.
(b) Test the same claim at the 0.05 significance level.
7. AM vs PM Scores: There are several sections of statistics, some in the morning (AM) and some in the afternoon (PM). We want to see if afternoon sections do better. We randomly select 22 students from the AM sections and 30 students from the PM sections. Their final averages (out of 100) are given in the table with other relevant statistics. The degrees of freedom (d.f.) is given to save calculation time if you are not using software.
The Test: Test the claim that the average for all students in the PM sections is greater than the AM sections. Use a 0.05 significance level.

|  | $n$ | $\bar{x}$ | $s^{2}$ | $s$ |
| :---: | :---: | :---: | :---: | :---: |
| AM | 22 | 71.2 | 250.3 | 15.82 |
| PM | 30 | 75.1 | 277.5 | 16.66 |
| degrees of freedom: d.f. $=\mathbf{4 7}$ |  |  |  |  |

8. Sleep by Med Students (Two-Tail \& One-Tail Tests): Here we consider the sleep habits of med students versus non-med students. The study consists of the hours of sleep per day obtained from 25 med students and 30 nonmed students. The summarized data is given in the table. Here, $\bar{x}$ is the mean hours of sleep per day from each sample. The degrees of freedom (d.f.) is given to save calculation time if you are not using software.

Mean Sleep Per Night Med Students vs Non-Med Students

| Student Type | $n$ | $\bar{x}$ | $s^{2}$ | $s$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Med $\left(x_{1}\right)$ | 25 | 5.7 | 0.9 | 0.95 |  |
| Non-Med $\left(x_{2}\right)$ | 30 | 6.3 | 1.9 | 1.38 |  |
| degrees of freedom: d.f. $=\mathbf{5 1}$ |  |  |  |  |  |

(a) The Two-Tailed Test: Test the claim that the mean hours of sleep for med and non-med students is different. Use a 0.05 significance level.
(b) The One-Tailed Test: Test the claim that, on average, med students get less sleep than non-med students. Use a 0.05 significance level.
(c) How can you explain the nearly contradictory conclusions from the previous two tests.
9.* Easier Professor - Significance Test: Next term, there are two sections of STAT 260 - Research Methods being offered. One is taught by Professor Smith and the other by Professor Jones. Last term, the average from Professor Smith's section was higher. You want to test whether or not the difference was significant. A significant difference is one that is not likely to be a result of random variation. You have the relevant data from last year and the results are summarized in the table. Here, the $\bar{x}$ 's are actually population means but we treat them like sample means. The degrees of freedom (d.f.) is given to save calculation time if you are not using software.
The Test: Test the claim that the average from Prof Smith's section was significantly different from Prof Jones' section. Use a 0.05 significance level.

| Professor | $n$ | $\bar{x}$ | $s^{2}$ | $s$ |
| :--- | :---: | :---: | ---: | ---: |
| Smith $\left(x_{1}\right)$ | 22 | 80.1 | 127.0 | 11.27 |
| Jones $\left(x_{2}\right)$ | 28 | 76.9 | 92.9 | 9.64 |
| degrees of freedom: d.f. $=\mathbf{4 1}$ |  |  |  |  |

10. Rainy Weekends - Significance Test: In the summer of 2012, the mean amount of rainfall on weekends at Acadia National Park was greater than the mean on weekdays. We want to test whether or not the average was significantly greater on weekends. The results are summarized in the table below where the $\bar{x}$ 's are actually population means but we treat them like sample means. These means represent the average daily rainfall in inches. The degrees of freedom (d.f.) is given to save calculation time if you are not using software.

The Test: Test the claim that the average rainfall on weekends was significantly greater than weekdays. Use a 0.05 significance level.
Summer 2012 - Acadia National Park

|  | $n$ | $\bar{x}$ | $s^{2}$ | $s$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Weekends | 36 | 0.252 | 0.283 | 0.532 |  |
| Weekdays | 85 | 0.151 | 0.284 | 0.533 |  |
| degrees of freedom: d.f. $=\mathbf{6 6}$ |  |  |  |  |  |

## Tests for Two Proportions (9.3)

11.* Boomerang Generation: The term 'Boomerang Generation' refers to the recent generation of young adults who have had to move back in with their parents.In a 2012 survey, 194 out of 808 randomly selected young adults (ages 18-34) had to move back in with their parents after living alone. In a similar survey from the year 2000, 288 out of 1824 young adults had to move back in with their parents. The table below summarizes this information. The standard error (SE) is given to save calculation time.

| year | total \# who <br> moved back $(x)$ | total \# in <br> survey $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| 2012 | 194 | 808 | 0.24010 |
| 2000 | 288 | 1824 | 0.15789 |
| Standard Error: SE $=\mathbf{0 . 0 1 6 3 4}$ |  |  |  |

(a) Test the claim that a greater proportion of all young adults moved back in with their parents in 2012 than in 2000. Test this claim at the 0.05 significance level.
(b) Can you support the claim at the 0.01 significance level?
12. Boomerang Generation - Short Term: In a 2010 Pew Research Center survey, 86 out of 390 randomly selected young adults (ages 18-34) had to move back in with their parents after living alone. In a 2012 survey, 194 out of 808 young adults had to move back in with their parents. The table below summarizes this information. The standard error (SE) is given to save calculation time.

| year | total \# who <br> moved back $(x)$ | total \# in <br> survey $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| 2012 | 194 | 808 | 0.24010 |
| 2010 | 86 | 390 | 0.22051 |
| Standard Error: SE $=\mathbf{0 . 0 2 6 0 9}$ |  |  |  |

The Test: Test the claim that a greater proportion of all young adults moved back in with their parents in 2012 than in 2010. Test this claim at the 0.05 significance level.
13.* Home vs Road Wins - Significance Test: For the 2011-2012 NHL regular season, the Chicago Blackhawks won 27 out of 41 home games and won 18 out of 41 away games. Clearly the Blackhawks won a greater proportion of home games. Here we investigate whether or not they did significantly better at home than on the road. The table summarizes the relevant data. Here, the $\hat{p}$ 's are population proportions but you should treat them as sample proportions. The standard error (SE) is given to save calculation time.

|  | total \# of <br> wins $(x)$ | total \# of <br> games $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| Home games | 27 | 41 | 0.65854 |
| Road games | 18 | 41 | 0.43902 |
| Standard Error: |  |  | SE $=\mathbf{0 . 1 0 9 9 0}$ |

(a) Test the claim that the proportion of wins at home is significantly greater than on the road. Test this claim at the 0.05 significance level.
(b) Can you support the claim at the 0.01 significance level?
14. Absentee rates - Friday vs Wednesday: We want to test whether or not more students are absent on Friday afternoon classes than on Wednesday afternoon classes. In a random sample of 300 students with Friday afternoon classes, 62 missed the class. In a different random sample of 300 students with Wednesday afternoon classes, 23 missed the class. The table below summarizes this information. The standard error (SE) is given to save calculation time if you are not using software.

| Class Day | total \# of <br> absences $(x)$ | total \# of <br> students $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| Friday | 62 | 300 | 0.20667 |
| Wednesday | 23 | 300 | 0.07667 |
| Standard Error: |  | SE $=\mathbf{0 . 0 2 8 4 7}$ |  |

(a) Test the claim that the absentee rate on all Friday afternoon classes is greater than the absentee rate on all Wednesday afternoon classes. Test this claim at the 0.05 significance level.
(b) Can you support the claim at the 0.01 significance level?
15.* Gun Murders - Texas vs New York - Significance Test: ${ }^{\mathbb{I}}$ In 2011, New York had much stricter gun laws than Texas. For that year, the proportion of gun murders in Texas was greater than in New York. Here we test whether or not the proportion was significantly greater in Texas. The table below gives relevant information. Here, the $\hat{p}$ 's are population proportions but you should treat them as sample proportions. The standard error (SE) is given to save calculation time if you are not using software.

| State | total \# of <br> gun murders $(x)$ | total \# of <br> murders $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| Texas | 699 | 1089 | 0.64187 |
| New York | 445 | 774 | 0.57494 |
| Standard Error: |  |  |  |
| SE $=\mathbf{0 . 0 2 2 8 9}$ |  |  |  |

(a) Test the claim that the proportion of murders committed with a gun was significantly greater in Texas than New York in 2011. Test this claim at the 0.05 significance level.
(b) Can you support the claim at the 0.01 significance level?
16. Gun Murders - Texas vs California - Significance Test: \| In 2011, California had much stricter gun laws than Texas. However, the proportion of gun murders in Texas was less than California. Here we test whether or not the proportion was significantly smaller in Texas. The table below summarizes this information. Here, the $\hat{p}$ 'are population proportions but you should treat them as a sample proportions. The standard error (SE) is given to save calculation time if you are not using software.

| State | total \# of <br> gun murders $(x)$ | total \# of <br> murders $(n)$ | proportion <br> $\hat{p}=x / n$ |
| :---: | :---: | :---: | :---: |
| Texas | 699 | 1089 | 0.64187 |
| California | 1220 | 1790 | 0.68156 |
| Standard Error: SE $=\mathbf{0 . 0 1 8 1 2}$ |  |  |  |

(a) Test the claim that the proportion of murders committed with a gun was significantly smaller in Texas than California in 2011. Test this claim at the 0.05
(b) Can you support the claim at the 0.01 significance level?

[^0]
[^0]:    ${ }^{\top}$ New York and Texas were chosen because they are both large states with large populations, a lot of murders, a large urban population, and very different gun laws.
    ${ }^{\|}$California and Texas were chosen for the same reasons.

